UNIT IV

INDUCTION MOTORS

Output equation of Induction motor – Main dimensions – Length of air gap- Rules for selecting rotor slots of squirrel cage machines – Design of rotor bars & slots – Design of end rings – Design of wound rotor -– Magnetic leakage calculations – Leakage reactance of polyphase machines-Magnetizing current - Short circuit current – Circle diagram - Operating characteristics.

OUTPUT EQUATION

Consider an 'm' phase machine having one circuit (parallel path) per phase, kVA rating of machine.

$$Q = \text{number of phase} \times \text{output voltage per phase} \times \text{current per phase} \times 10^{-3}$$
$$= m\, E_{ph}\, I_{ph} \times 10^{-3}.$$

Terminal voltage of each phases may be taken equal to the induced emf per phase. We have,

Induced emf per phase $E_{ph} = 4.44\, f\, \Phi\, T_{ph}\, K_w.$ $\therefore$ $Q = m \times 4.44\, f\, \Phi\, T_{ph} K_w + I_{ph} \times 10^{-3}$

But $\qquad\qquad f = pn_s/2$

Therefore we can write,

$$Q = m \times 4.44\, (pn_s/2)\, \Phi\, T_{ph}\, K_w\, I_{ph} \times 10^{-3}$$
$$= 1.11\, K_w\, (p\Phi)\, 2m\, I_{ph},\, T_{ph})n_s \times 10^{-3}$$

Now current in each conductor $I_z = I_{ph}$ (as there is only one circuit per phase).

Total number of armature conductors

$$Z = \text{number of phases} \times (2 \times \text{turns per phase}) = 2\, m\, T_{ph}.$$

$\therefore$ Total electric loading $= I_z\, Z = 2\, m\, I_{ph}\, T_{ph}.$

Hence, $\qquad\qquad Q = 1.11\, K_w\, (p\Phi)\, (I_z\, Z)n_s \times 10^{-3}$

$$= 1.11\, K_w\, (\text{total magnetic loading}) (\text{total electric loading})$$
$$(\text{synchronous speed} \times 10^{-3})$$

But $\qquad\qquad p\Phi = \pi DL\, B_{av}$ and $I_z\, Z = \pi D\, ac$

Substituting these values in Eqn. 6.16, we have

$$Q = (1.11\, K_w\, (\pi DL\, B_{av})\, (\pi D\, ac)n_s \times 10^{-3}$$

$$= (1.11\, \pi^2\, B_{av}\, ac\, K_w \times 10^{-3})\, D^2\, Ln_s$$
$$= (11\, B_{av}\, ac\, K_w \times 10^{-3})\, D^2\, Ln_s$$
$$= C_0\, D^2L\, n_s$$

$$\boxed{Q = C_0\, D^2L\, n_s}$$ -- (1)

where $C_0 = 11\, B_{av}\, ac\, K_w \times 10^{-3}$

Equ. (1) is known as the output equation of an a.c. machine. Quantity C_0 is called the output co-efficient.

MAIN DIMENSIONS

The armature diameter D (or stator bore) and stator core length L are known as main dimensions. The quantity D^2L is in m^3 and called as volume of the machine.
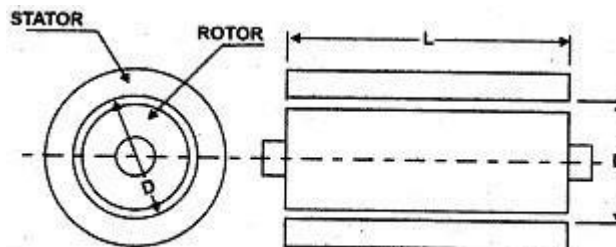

Fig.   Main Dimensions D and L

The size or volume of the active parts of machine as given by D^2L depends upon two factors.(i) output co–efficient, C_o (ii) the speed, n_s. For higher values of C_o and n_s, D^2L and the size of the machine decreases. Thus, to obtain smallest dimensions of the machine, the output co–efficient C_o must be selected as highest as possible. The separation of D and L, from the product D^2L, depends on the ratio L/τ, where $\tau = \pi D/p$. In induction motors, most of the operating characteristics are decided by L/τ ratio of the motor. The ratio of core length to pole pitch (L/τ) for various design features are listed in Table

Table   L/τ values:

Design Feature	Ratio L/τ
(1) Minimum cost	1.5 to 2
(2) Good Power Factor	1 to 1.25
(3) Good Efficiency	1.5
(4) Good Overall design	1

Generally L/τ lies between 0.6 to 2. It can be shown that for best power factor the pole pitch τ is given by the equation $\tau = \sqrt{0.18L}$

The diameter of the stator bore and hence the diameter of the rotor is also limited by peripheral speed. Standard constructions are employed for peripheral speeds upto 60 m/s. For higher peripheral speeds upto 75 m/s, special construction methods should be employed for rotor which results in higher cost. For a normal design, the diameters should be so chosen that the peripheral speed does not exceed about 30 m/sec. The stator is provided with radial ventilating ducts if the core length exceeds 125 mm. The width of each duct is about 8 to 10 mm.

CHOICE OF SPECIFIC LOADINGS

1. Choice of specific magneticloadings

(i) *Power factor.* The value of flux density in air gap should be small as otherwise the machine will draw a large magnetising current giving a poor power factor. However, in induction motors the flux density in the air gap, should be such that there is no saturation in any part of the magnetic circuit.

(ii) *Iron loss.* An increased value of gap density results in increased iron loss and decreased efficiency.

(iii) *Overload capacity.* The value of air gap flux density determines the overload capacity. A high value of B_{av} means that the flux per pole is large. Thus for the same voltage, the winding requires less turns per phase and if the number of turns is less, the leakage reactance becomes small. With small leakage reactance the circle diagram of the machine has a large diameter which means that the maximum output, which the machine is capable of giving, is large or in other words the machine has a large *over-load capacity*. Thus, with the assumption of a higher value of B_{av}, we get a higher value of overload capacity.

For 50 Hz machines of normal design the value of B_{av} lies between 0.3 and 0.6 Wb/m². For machines used in cranes, rolling mills etc., where a large overload capacity is required, a value of 0.65 Wb/m² may be used.

2. Choice of specific electricloadings

(i) *Copper loss and temperature rise.* A large value of *ac* means that a greater amount of copper is employed in the machine. This results in higher copper losses and large temperature rise of embedded conductors

(ii) *Voltage.* A small value of *ac* should be taken for high voltage machines as in their case the space required for insulation is large

(iii) *Overload capacity.* A large value of ampere conductors would result in large number of turns per phase. This would mean that the leakage reactance of the machine becomes high and the diameter of circle diagram is reduced resulting in reduced value of overload capacity. Therefore, higher the value of *ac*, the lower would be the overload capacity.

Hence the value of ampere conductors per metre depends upon the size of the motor, the voltage of stator winding, the type of ventilation and the overload capacity desired. It varies between 5000 to 45000 ampere conductors per metre depending upon the factors listed above.

DESIGN OF STATOR

1. Statorturns/phase

Turns/phase $\quad T_S = \dfrac{E_S}{4.44 \phi_m f K_{WS}} \qquad$ and $\phi_m = \dfrac{B_{av} \pi DL}{P}$

2. Area of statorconductors

Area of each stator conductor $a_s = \dfrac{I_S}{\delta_s}$

Stator current per phase $\quad I_S = \dfrac{\text{Input kVA}}{3E}$

Current density in stator $\delta_s = 3$ to 5 A/mm²

3. Shape of Stator slots

Slots may be completely open or semi closed. Semi closed slots are preferred for induction motors because with their use the gap contraction factor is small giving a small value of magnetizing current. The use of semi enclosed slots results in low tooth pulsation loss and a much quieter operation as compared with that with open slots.

4. Stator slots

Number of stator slots $S_S = 3 p q_s$
   Where p = Number of poles
       q_S = Number of stator slots/pole/phase ≥ 2

Number of stator slots $S_s = \dfrac{\pi D}{y_{ss}}$

Where y_{ss} = stator slot pitch

= 10 to 15 mm   for single layer winding

= 15 to 25 mm   for double layer winding

5. Statorconductors

Number of stator conductors

$Z_S = 2\ T_S$ for single phase

$Z_S = 6T_S$ for three phase

Number of stator conductor per slot

$Z_{ss} = \dfrac{Z_S}{S_s}$

= Integer for single layer winding

= Even integer for double layer winding

6. Slotloading

Slot loading $= I_Z\ Z_{SS}$ and $I_Z = I_S$

NOTE: 1. The stator is provided with radial ventilating ducts if the core length exceeds 100 to 125 mm.

2. The width of each duct is about 8 to 10 mm.

3. Input kVA $Q = \dfrac{kW}{\eta\ \cos\Phi}$

4. Input kVA $Q = \dfrac{hp \times 0.746}{\eta\ \cos\Phi}$

5. $K_{ws} = 0.955$

EXAMPLE: 01

Determine the approximate diameter and length of stator core, the number of stator slots and the number of stator conductors for a 11 kW, 400V, 3ϕ, 4-pole, 1425 rpm, delta connected induction motor. B_{av} = 0.45 Wb/m², ac = 23000 amp. cond./m, full load efficiency = 0.85, pf = 0.88, L/τ = 1. The stator employs a double layer winding.

Given Data

11 kW	delta connected	1425 rpm
3ϕ	double layer winding	B_{av} = 0.45 Wb/m²
4 - pole	ac = 23000 amp.cond./m.	pf = 0.88
400 V	η = 0.85	L/τ =1

Solution

$$\text{kVA input} = \frac{\text{Output}}{\eta \times \text{pf}} = \frac{11}{0.85 \times 0.88} = 14.7 \text{ kVA}$$

$$\text{Synchronous speed, } n_s = \frac{2f}{p} = \frac{2 \times 50}{4} = 25 \text{ rps}$$

Let, K_{ws} = 0.955

$$C_o = 11 \, K_{ws} B_{av} \text{ac} \times 10^{-3}$$
$$= 11 \times 0.955 \times 0.45 \times 23000 \times 10^{-3} = 108.7268 \text{ kVA / m}^3 \text{ - rps}$$

$$\text{kVA input, } Q = C_o \, D^2 \, L \, n_s$$

$$\therefore D^2 L = \frac{Q}{C_o n_s} = \frac{14.7}{108.7268 \times 25} = 0.0054 \text{ m}^3$$

$$\text{Given that, } L/\tau = 1 \quad \therefore L = \tau = \frac{\pi D}{p}$$

$$\text{Put, } L = \frac{\pi D}{p} \text{ in the equation for D}^2 L$$

$$\therefore D^2 L = D^2 \frac{(\pi D)}{p} = 0.0054 \quad \text{or} \quad D^3 \frac{\pi}{p} = 0.0054$$

$$\therefore D = \left(\frac{0.0054 \times p}{\pi} \right)^{1/3} = 0.1902 \text{ m}$$

$$L = \frac{\pi D}{p} = \frac{\pi \times 0.1902}{4} = 0.1494 \text{ m.}$$

$$\boxed{D = 0.19 \text{ m} \quad \text{and} \quad L = 0.15 \text{ m}}$$

$$\text{Maximum flux per pole,} \phi_m = \frac{B_{av} \pi D L}{p} = \frac{0.45 \times \pi \times 0.19 \times 0.15}{4} = 0.01 \text{ Wb}$$

Since the stator is delta connected, the line voltage is same as phase voltage.

$$\text{Stator turns per phase, } T_s = \frac{E_s}{4.44 \, f \phi_m K_{ws}}$$

$$= \frac{400}{4.44 \times 50 \times 0.01 \times 0.955} = 188$$

The stator slots should be multiple of q, where q is slots per pole per phase.

Stator slots, S_s = Number of phaes × Poles × q

For q = 2, S_s = 3 × 4 × 2 = 24
For q = 3, S_s = 3 × 4 × 3 = 36
For q = 4, S_s = 3 × 4 × 4 = 48

The stator slot pitch should lie between 15 mm to 25 mm.

When, S_s = 36, $y_{ss} = \frac{\pi D}{S_s} = \frac{\pi \times 0.19 \times 10^3}{36} = 16.58$ mm

When S_s = 36, the slot pitch (y_{ss}) lies between 15 to 25 mm. Hence the stator slots can be 36.

Conductors per slot, $Z_{ss} = \frac{6T_s}{S_s} = \frac{6 \times 188}{36} = 31.333$

Z_{ss} should be even integer for double layer winding and so it is 30 or 32.

Let, Z_{ss} = 32, Total stator conductors = $S_s \times Z_{ss}$ = 36 × 32 = 1152

New value of turns per phase, $T_s = \frac{Z_{ss} S_s}{6} = \frac{32 \times 36}{6} = 192$

Results

Diameter of stator	=	0.19 m
Length of stator	=	0.15 m
Number of stator slots	=	36
Total stator conductor	=	1152
Turns per phase	=	192

EXAMPLE: 02

Estimate the stator core dimensions, number of stator slots and number of stator conductors per slot for a 100 kW, 3300V, 50Hz, 12 pole, star connected slip ring induction motor. B_{av} = 0.4 Wb/m², ac = 25000 amp.cond./m, η = 0.9, pf = 0.9. Choose main dimensions to give best power factor. The slot loading should not exceed 500 amp. conductors.

Given Data

100 kW	3300 V	B_{av} = 0.4 Wb/m²
50 Hz	12 pole	ac = 25000 amp.cond./m.
η = 0.9	pf = 0.9	slot loading ≤ 500 amp.cond.
star connected	3 phase	

Solution

kVA input, $Q = \frac{\text{output}}{\eta \times \text{pf}} = \frac{100}{0.9 \times 0.9} = 123.457$ kVA

Let, K_{ws} = 0.96

Output coefficient, $C_0 = 11 \, B_{av} \, \text{ac} \, K_{ws} \times 10^{-3}$

$$= 11 \times 0.4 \times 25000 \times 0.96 \times 10^{-3}$$

$$= 105.6 \text{ kVA/m}^3\text{-rps}$$

Synchronous speed, $n_s = \dfrac{2f}{p} = \dfrac{2 \times 50}{12} = 8.33$ rps

We know that, kVA input, $Q = C_o \, D^2 \, L \, n_s$

$$\therefore D^2 L = \frac{Q}{C_o n_s} = \frac{123.457}{105.6 \times 8.33} = 0.1403 \ m^3$$

For best power factor, $\tau = \sqrt{0.18 \, L}$

But, $\tau = \dfrac{\pi D}{p}$, $\qquad \therefore \dfrac{\pi D}{p} = \sqrt{0.18 \, L}$

On squaring we get, $\dfrac{\pi^2 D^2}{p^2} = 0.18L$

$$\therefore D^2 = \frac{0.18 \times p^2}{\pi^2} L = \frac{0.18 \times 12^2}{\pi^2} L = 2.6262 \, L$$

Put, $D^2 = 2.6262 \, L$ in the equation for $D^2 \, L$

$\therefore D^2 L = 2.6262 \, L \times L = 0.1403$

$$L = \sqrt{\frac{0.1403}{2.6262}} = 0.2311 \ m \approx 0.23 \ m$$

$D^2 = 2.6262 \, L$, $\qquad \therefore D = \sqrt{2.6262 \times 0.23} = 0.7772 \ m \approx 0.78 \ m$

$$\boxed{L = 0.23 \ m \qquad \text{and} \qquad D = 0.78 \ m}$$

Since the stator is star connected,

Stator voltage per phase, $E_s = \dfrac{3300}{\sqrt{3}} = 1905.256$ V

Flux per pole, $\phi_m = \dfrac{B_{av} \pi D L}{p} = \dfrac{0.4 \times \pi \times 0.78 \times 0.23}{12} = 0.0188$ Wb

Stator turns per phase, $T_s = \dfrac{E_s}{4.44 \, f \, \phi_m K_{ws}} = \dfrac{1905.256}{4.44 \times 50 \times 0.0188 \times 0.96} = 478$

The stator slot pitch should lie between 15 to 25 mm.

Stator slots, $S_s = \dfrac{\pi D}{y_{ss}}$

When $y_{ss} = 15\text{mm}$, $S_s = \dfrac{\pi \times 0.78}{15 \times 10^{-3}} = 163$ $\qquad$ When $y_{ss} = 25\text{mm}$, $S_s = \dfrac{\pi \times 0.78}{25 \times 10^{-3}} = 98$

The stator slots, S_s should lie between 98 to 163.

The stator slots be multiple of q, where q is slot per pole per phase.

Stator slots, S_s = Number of phases × poles × q

When, q = 2, $S_s = 3 \times 12 \times 2 = 72$
When, q = 3, $S_s = 3 \times 12 \times 3 = 108$

The S_s values of 108 and 144 lie in the range of 98 to 163.

$\therefore$ S_s can be either 108 or 144.

Check for slot loading

$$\text{Stator current per phase} = \frac{kVA \times 10^3}{\sqrt{3} \times V_L} = \frac{123.457 \times 10^3}{\sqrt{3} \times 3300} = 21.6 \text{ A}$$

(since star connected, $I_L = I_{ph}$)

When $S_s = 108$

$$Z_{ss} = \frac{6T_s}{S_s} = \frac{6 \times 478}{108} = 26.55 \approx 26$$

Slot loading $= Z_{ss}I_s = 26 \times 21.6$
                $= 561.6$ amp.cond.

When $S_s = 144$,

$$Z_{ss} = \frac{6T_s}{S_s} = \frac{6 \times 478}{144} = 19.91 \approx 20$$

Slot loading $= Z_{ss}I_s = 20 \times 21.6$
                $= 432$ amp.cond.

When $S_s = 144$, the slot loading does not exceeds 500 amp.cond. Hence 144 slots is suitable for the machine.

$$\text{Total stator conductors} = S_s \times Z_{ss} = 144 \times 20 = 2880$$

$$\text{New value of turns per phase, } T_s = \frac{Z_{ss} \, S_s}{6} = \frac{20 \times 144}{6} = 480$$

Result

Diameter of stator	=	0.78 m
Length of stator	=	0.23 m
Number of stator slots	=	144
Total stator conductors	=	2880
Turns per phase	=	480

EXAMPLE: 03

Determine the D and L of a 70 HP, 415V, 3-phase, 50 Hz, star connected, 6 pole induction motor for which **ac** = 30000 amp.cond./m and $B_{av} = 0.51$ Wb/m². Take $\eta = 90$ % and pf = 0.91. Assume $\tau = L$. Estimate the number of stator conductors required for a winding in which the conductors are connected in 2- parallel paths. Choose a suitable number of conductors per slots, so that the slot loading does not exceed 750 amp. cond.

Given Data

70 HP	415 V	$B_{av} = 0.51$ Wb/m²
3 - phase	50 Hz	ac = 30000 amp.cond./m
$\eta = 0.9$	pf = 0.91	star connected
6-pole	$\tau = L$	slot loading ≤ 750 amp.cond.

conductors are connected in 2 - parallel paths.

Solution

$$\text{kVA input, } Q = \frac{HP \times 0.746}{\eta \times pf} = \frac{70 \times 0.746}{0.9 \times 0.91} = 63.76 \text{ kVA}$$

$$\text{Output coefficient, } C_o = 11 \, B_{av} \, \text{ac} \, K_{ws} \times 10^{-3}$$

$$= 11 \times 0.51 \times 30000 \times 0.955 \times 10^{-3}$$

$$= 160.7265 \text{ kVA}/m^3 - rps$$

$$\text{Synchronous speed, } n_s = \frac{2f}{p} = \frac{2 \times 50}{6} = 16.667 \text{ rps}$$

We know that, $Q = C_o D^2 L n_s$

$$\therefore D^2 L = \frac{Q}{C_o n_s} = \frac{63.76}{160.7265 \times 16.667} = 0.0238 \text{ m}^3$$

Given that, $\tau = L$	Put, $L = 0.5236 \text{ D}$ in the equation for $D^2 L$
But $\tau = \dfrac{\pi D}{p}$	$\therefore D^2 L = 0.0238$
$\therefore L = \tau = (\pi D / p)$	$D^2 (0.5236 D) = 0.0238$
$\quad = (\pi D / 6)$	$D = \left(\dfrac{0.0238}{0.5236}\right)^{1/3}$
$\quad = 0.5236 \text{ D}$	$D = 0.35688 \text{ m} \approx 0.36 \text{ m.}$

$$\boxed{L = 0.5236 \quad ; \quad D = 0.5236 \times 0.36 = 0.1885 \approx 0.19 \text{ m.}}$$

$$\text{Flux per pole, } \phi_m = \frac{B_{av} \pi DL}{p} = \frac{0.51 \times \pi \times 0.36 \times 0.19}{6} = 0.0183 \text{ Wb}$$

$$\text{Turns per phase, } T_s = \frac{E_s}{4.44 \, f \, \phi_m K_{ws}} = \frac{415/\sqrt{3}}{4.44 \times 50 \times 0.0183 \times 0.955}$$

$$= 61.756 \approx 62$$

Since the conductors are placed in two parallel paths,

Total stator conductor $= 6T_s \times 2 = 12 \, T_s = 12 \times 62 = 744$ conductors

The slot pitch y_{ss} should lie between 15 to 25 mm

When $y_{ss} = 15$mm	When $y_{ss} = 25$mm
$S_s = \dfrac{\pi D}{y_{ss}} = \dfrac{\pi \times 0.36}{15 \times 10^{-3}} = 75$	$S_s = \dfrac{\pi D}{y_{ss}} = \dfrac{\pi \times 0.36}{25 \times 10^{-3}} = 45$

The number of stator slots lie in the range of 45 to 75.

The stator slots should be multiple of q where q is slots per pole per phase.

Stator slots, S_s = Number of phases × poles × q

When $\quad q = 2, \quad S_s = 3 \times 6 \times 2 = 36$
When $\quad q = 3, \quad S_s = 3 \times 6 \times 3 = 54$
When $\quad q = 4, \quad S_s = 3 \times 6 \times 4 = 72$

The values of S_s which lies between 45 to 75 are $S_s = 54$ and $S_s = 72$.

$$\text{Stator current per phase, } I_s = \frac{kVA \times 10^3}{\sqrt{3} V_L} = \frac{63.76 \times 10^3}{\sqrt{3} \times 415} = 88.7A$$

$$\left(\text{stator is star connected, } \therefore I_L = I_p\right)$$

$$\text{Current through conductor, } I_z = \frac{I_s}{a} = \frac{88.7}{2} = 44.35A$$

(conductors are connected in two parallel paths)

Check for slot loading

When $S_S = 54$

$$\left.\begin{array}{c}\text{Conductors}\\\text{per slot}\end{array}\right\} Z_{ss} = \frac{744}{54} = 13.77 \approx 14$$

Slot loading $= Z_{ss}I_z = 14 \times 44.35$
$\qquad\qquad = 620.9$ amp.cond.

When $S_S = 72$

$$\left.\begin{array}{c}\text{Conductors}\\\text{per slot}\end{array}\right\} Z_{ss} = \frac{744}{72} = 10.33 \approx 11$$

Slot loading $= Z_{ss}I_z = 11 \times 44.35$
$\qquad\qquad = 487.85$ amp.cond.

In both the cases slot loading is not exceeded.

For lower fabrication cost $S_s = 54$

For low temperature rise $S_s = 72$

Let $S_s = 54$

$\therefore Z_{ss} = 14$

Total stator conductors $= Z_{ss} \times S_s = 14 \times 54 = 756$ conductors

New value of turns per phase, $T_s = \dfrac{Z_{ss} \times S_s}{6 \times 2} = \dfrac{756}{6 \times 2} = 63$

Result

Diameter of stator	=	0.36 m
Length of rotor	=	0.19 m
Turns per phase	=	63
Number of stator slots	=	54
Conductors per slot	=	14

EXAMPLE: 04

Determine the main dimensions, number of radial ventilating ducts, number of stator slots and the number of turns per phase of a 3.7 kW, 400 volt, 3 phase, 4 pole, 5 Hz squirrel cage induction-motor to be started by a star delta starter. Work out the winding details.

Assume :

Average flux density in the gap = 0.45 Wb/m², ampere conductors per metre = 23000, efficiency = 0.85, and pwer factor 0.84.

Machines rated at 3.7 kW, 4 pole are sold at a competitive price and therefore choose the main-dimensions to give a cheap design.

Assume :

winding factor = 0.955, stacking factor = 0.9.

GIVEN DATA

P=3.7 kW   V=400 V   3phase   p=4   f=50 Hz   Squirrel cage IM B_{av}=0.45 Wb/m²
Ac=23000 η=0.84   design machine for minimum cost K_w=0.955 K_i=0.9
Started by star-delta starter $\eta = 0.85$

Solution.

(i) Main Dimensions

$$\text{kVA input } Q = \frac{\text{kW}}{\eta \cos \phi} = \frac{3.7}{0.85 \times 0.84} = 5.18$$

Output co-efficient $\quad C_0 = 11 K_w B_{av} ac \, 10^{-3}$

$\qquad\qquad\qquad\qquad = 11 \times 0.955 \times 23000 \times 10^{-3} = 108.7$

Synchronous speed $n_s = 2\ f/p = 2 \times 50/4 = 25$ r.p.s.

$\therefore$ Product $\qquad D^2L = \dfrac{Q}{C_0 n_s} = \dfrac{5.18}{(108.7 \times 25)} = 1.91 \times 10^{-3}$ m^3.

For a cheap design ratio, $L/\tau = 1.5$ to 2. Taking $L/\tau = 1.5$,

we have $\qquad \dfrac{L}{\pi D / p} = 1.5$ or $\dfrac{L}{D} = \dfrac{1.5 \times \pi}{p} = \dfrac{1.5 \times \pi}{4} = 1.178$.

Now $\qquad D^2L = 1.91 \times 10^{-3}$ m^3 or $1.178\ D^3 = 1.91 \times 10^{-3}$

or $\qquad D = 0.117$ m ≈ 0.12 m and $L = \dfrac{1.91 \times 10^{-3}}{(0.12)^2} \approx 0.13$ m.

$$\boxed{D = 0.12\ m \quad L = 0.13\ m}$$

Pole pitch $\qquad \tau = \pi D/p = \pi \times 0.12/4 = 0.094$ m.

The length of core is 0.13 m and therefore one radial duct 10 mm wide is provided.

$\therefore$ Net iron length $L_i = 0.9\ (0.13 - 0.01) = 0.108$ m.

(ii) Turns per phase

Flux per pole $\qquad \Phi_m = B_{av}\ L\ \tau = 0.45 \times 0.13 \times 0.094 = 5.5 \times 10^{-3}$ Wb.

As the machine is started by a star delta starter, it is designed for delta connection.

$\therefore$ Stator voltage per phase $E_s = 400$ V.

Stator turns per phase $T_s = \dfrac{E_s}{4.44\ f\ \Phi_m K_{ws}}$

$\qquad = \dfrac{400}{4.44 \times 50 \times 5.5 \times 10^{-3} \times 0.955} = 343$.

(iii) Number of stator slots. It is a small sized machine and since semi-enclosed slots are used for this machine the slot pitch can be lower than 15 mm. However, for mechanical reasons the slot pitch should not be below 10 mm.

Taking, slots per pole per phase $q_s = 3$.

Total number of stator slots $\qquad S_s = 3 \times 4 \times 3 = 36$.

Stator slot pitch $\qquad = y_{ss} = \pi D/S_s = \pi \times 0.12/36$

$\qquad\qquad = 0.01047$ m $= 10.47$ mm.

Total number of stator conductors $= 6 T_s = 6 \times 343 = 2058$

Conductors per slot $\qquad Z_{ss} = \dfrac{2058}{36} = 57$.

$\therefore$ Actual number of turns per phase $T_s = \dfrac{36 \times 57}{2 \times 3} = 342$.

(iv) Winding details. For small machines like this, a single layer mush winding placed in semi-enclosed slots is used. In a single layer winding, each coil occupies two slots and, therefore,

number of coils $\qquad = 36/2 = 18$

and number of coils per phase $\qquad = 18/3 = 6$.

EXAMPLE: 05

Find the main dimensions of a 15 kW, 3 phase, 400 V, 50 Hz, 2810 r.p.m. squirrel cage induction motor having an efficiency of 0.88 and a full load power factor of 0.9. Assume :

specific magnetic loading = 0.5 Wb/m^2 ; specific electric loading = 25000 A/m.

Take the rotor peripheral speed as approximately 20 m/s at synchronous speed.

GIVEN DATA

P=15 kW   3 phase   V=400 V   f=50 Hz   N=2180 rpm   η=0.88   cosΦ=0.9
B_{av}=0.5 Wb/m²   ac=25000 V_{cr}=20 m/s   Squirrel cage IM

Solution.

kVA input $\qquad Q = \dfrac{15}{0.88 \times 0.9} = 18.94.$

Output co-efficient $\quad C_0 = 11\, K_w\, B_{av}\, ac \times 10^{-3}$
$$= 11 \times 0.955 \times 0.5 \times 25000 \times 10^{-3} = 131.3$$

The speed of the rotor at full load is 2810 r.p.m. and the nearest synchronous speed corresponding to 50 Hz is 3000 r.p.m.

Synchronous speed $\quad n_s = 3000/60 = 50$ r.p.s.

∴ Product $\qquad D^2L = \dfrac{Q}{C_0 n_s} = \dfrac{18.94}{131.3 \times 50} = 2.88 \times 10^{-3}$ m³.

The rotor diameter in an induction motor is almost equal to stator bore.

∴ $\qquad \pi D n_s = 20$

or $\qquad D = \dfrac{20}{\pi \times 50} = 0.1257$ m,

and $\qquad L = \dfrac{2.88 \times 10^{-3}}{(0.1275)^2} = 0.177$ m.

$$\boxed{D = 0.1257 \text{ m} \quad L = 0.177 \text{ m}}$$

EXAMPLE: 06

Determine the main dimensions, turns per phase, number of slots, conductor cross-section and slot area of a 250 h.p., 3 phase, 50 Hz, 400 V, 1410 r.p.m. slip ring induction motor. Assume B_{av} = 0.5 Wb/m², ac = 30000 A/m, efficiency = 0.9 and power factor = 0.9, winding factor = 0.955, current density = 3.5 A/mm². The slot space factor is 0.4 and the ratio of core length to pole pitch is 1.2. The machine is delta connected.

GIVEN DATA

H.P=250   3 phase   V=400 V   N=1410 rpm   Slip ring IM
B_{av}=0.5 Wb/m²   ac=30000 A/m   η=0.9   cosΦ=0.9 K_w=0.955
δ=3.5 A/mm² S_f=0.4   L/τ=1.2   f=50 Hz
Delta connected machine

(i) Main Dimensions

The speed of motor is 1410 r.p.m. and the nearest synchronous speed corresponding to 50 Hz is 1500 r.p.m.

Synchronous speed $\quad n_s = 1500/60 = 25$ r.p.s.
Number of poles $\qquad = 2 \times 50/25 = 4.$
Output co-efficient $\quad C_0 = 11 \times 0.955 \times 0.5 \times 30,000 \times 10^{-3} = 157.6$

kVA input $\qquad Q = \dfrac{250 \times 0.746}{0.9 \times 0.9} = 230.2$

∴ $\qquad D^2L = \dfrac{Q}{C_0 n_s} = \dfrac{230.2}{157.5 \times 25} = 58.4 \times 10^{-3}$ m³.

We have $\qquad L/\tau = 1.2$ or $L/D = 1.2 \times \pi/4 = 0.942.$

∴ $\qquad 0.942\, D^3 = 58.4 \times 10^{-3}$

or $\qquad D = 0.395$ m and $L \approx 0.375$ m

(ii) Winding

Flux per pole $\quad \Phi_m = 0.5 \times \pi \times 0.395 \times 0.375/4 = 582 \times 10^{-3}$ Wb

The machine is delta connected.

$\therefore$ Stator voltage per phase $\quad E_s = 400$ V.

$\therefore$ Stator turns per phase $\quad T_s = \dfrac{400}{4.44 \times 50 \times 58.2 \times 10^{-3} \times 0.955} = 32.4$

Total conductors $\quad = 6T_s = 6 \times 32 = 192.$

The slot pitch lies between 15 to 25 mm.

$\therefore$ The number of slots lies between :

$$\frac{\pi \times 0.395 \times 10^3}{25} = 50 \text{ to } \frac{\pi \times 0.395 \times 10^3}{15} = 84$$

The machine is large in size and therefore a large number of slots should be chosen.

The value of number of slots per pole per phase and the conductors per slot should be so chosen that there is not much difference in the value of conductors provided and the conductors calculated earlier.

Taking 5 slots per pole per phase,

Total number of stator slots $\quad = 3 \times 4 \times 5 = 60$

Providing 3 conductors per slot. Total number of conductors $= 3 \times 60 = 180.$

Turns per phase $\quad = 180/6 = 30.$

The value of turns per phase calculated earlier is 32.4. Thus there is a decrease of about 7 per cent in the turns provided and therefore the value of flux density would increase by this amount.

Single layer concentric winding with semi-enclosed slots is used. (The number of conductors per slot is odd and therefore double layer winding is not possible).

Stator current per phase $\quad I_s = \dfrac{250 \times 746}{3 \times 400 \times 0.9 \times 0.9} = 192$ A.

Area of stator conductor $\quad a_s = I_s/\delta_s = 192/3.5 = 55$ mm^2.

Total copper area in each slot $\quad = 3 \times 55 = 165$ mm^2.

Total area of slot $\quad = \dfrac{\text{copper area per slot}}{\text{space factor}} = \dfrac{165}{0.4} = 412.5$ mm^2.

EXAMPLE: 07

A 15 kW, 440 V, 4 pole, 50 Hz, 3 phase induction motor is built with a stator bore 0.25 m and a core length of 0.16. The specific electric loading is 23000 ampere conductors per metre. Using the data of this machine, determine the core dimensions, number of stator slots and number of stator conductors for a 11 kW, 460 V, 6 pole, 50 Hz motor. Assume a full load efficiency of 84 per cent and power factor of 0.82 for each machine. The winding factor is 0.955.

GIVEN DATA

    (i) P=15 kW $\quad$ V=440 V $\quad$ p=4 $\quad$ f=50 Hz $\quad$ 3 phase $\quad$ D=0.25 m $\quad$ L=0.16m

        ac=23000 a/m

    (ii) P=11 kW $\quad$ V=460 V $\quad$ p=6 $\quad$ f=50 Hz $\quad$ η=0.84 $\quad$ cosΦ=0.82 $\quad$ K$_w$=0.955

15 kW Motor :

kVA input $= \dfrac{15}{0.84 \times 0.82} = 21.8.$ Synchronous speed $n_s = 2 \times 50/4 = 25$ r.p.s.

Now, $\quad C_0 = \dfrac{Q}{D^2 L n_s} = \dfrac{21.8}{(0.25)^2 \times 0.16 \times 25} = 87.2.$

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From Eqn.

$$C_0 = 11 K_w B_{av} \, ac \times 10^{-3} = 11 \times 0.955 \times B_{av} \times 23000 \times 10^{-3}$$
$$= 241.6 \, B_{av}$$

∴ Average flux density in the air gap $B_{av} = 87.2/241.6 = 0.36$ Wb/m².

Pole pitch $\qquad \tau = \pi \times 0.25/4 = 0.196$ m.

∴ Ratio $\qquad L/\tau = 0.16/0.196 = 0.815.$

1/kW Motor :

We have to use the same data for the 11 kW machine as is calculated above for 15 kW machine.

∴ $\qquad B_{av} = 0.36$ Wb/m² ; $ac = 23000$ A/m ;

$\qquad L/\tau = 0.815$ and $C_0 = 87.2.$

Synchronous speed $\quad n_s = \dfrac{2f}{p} = \dfrac{2 \times 50}{6} = 16.67$ r.p.s. kVA $= \dfrac{11}{0.84 \times 0.82} = 16.$

∴ Production $\qquad D^2 L = \dfrac{16}{87.2 \times 16.67} = 11 \times 10^{-3}$ m³

and ratio $\qquad \dfrac{L}{D} = 0.815 \times \dfrac{\pi}{6} = 0.427$

or $\qquad 0.427 \, D^3 = 11 \times 10^{-3}$

∴ $\qquad D = 0.30$ m and $L = 0.125$ m.

The number of stator slots lies between $\dfrac{\pi \times 0.3 \times 10^3}{25} = 37$ and $\dfrac{\pi \times 0.3 \times 10^3}{15} = 63.$

Using 3 slots per pole per phase,

Number of stator slots $\qquad S_s = 3 \times 6 \times 3 = 54.$

Flux per pole $\qquad \Phi_m = \dfrac{0.36 \times \pi \times 0.3 \times 0.125}{6} = 7.07 \times 10^{-3}$ Wb

Delta connection is used for the stator winding.

∴ Stator voltage per phase $\qquad E_s = 460$ V.

Stator turns per phase $\qquad T_s = \dfrac{460}{4.44 \times 50 \times 7.07 \times 10^{-3} \times 0.955} = 307.$

Total number of stator conductors $\qquad = 6 \times 307 = 1842.$

∴ Conductors per slot $\qquad Z_{ss} = \dfrac{1842}{54} = 34.1.$

Using 34 conductors per slot, total conductors $= 1836.$

∴ Stator turns per phase $\qquad T_s = 1836/6 = 306.$

LENGTH OF AIR GAP

The length of air gap in induction motor is decided by considering the following factors.

- Power factor
- Pulsation loss
- Cooling
- Over-load capacity
- Unbalanced magnetic pull
- Noise

Power factor

The mmf required to send the flux through air gap is proportional to the product of flux density and the length of air gap. Even with very small densities, the mmf required for air gap is much more than that for the rest of the magnetic circuit. Therefore, it is the length of air gap that primarily determines the magnetizing current drawn by the machine.

Overload capacity

The length of air gap affects the value of zig zag leakage reactance which forms a large part of total leakage reactance. If the length of air gap is large then the zig zag leakage flux will be less and so the leakage reactance will be less. With lesser value of leakage reactance the over load capacity increases. Hence, greater is the length of air gap, greater is the overload capacity.

Pulsation loss

With larger length of air gap, the variation of reluctance due to slotting is small. The tooth pulsation loss, which is produced due to variation in reluctance of the air gap, is reduced accordingly. Therefore, the pulsation loss is less with large air gaps.

Unbalanced magnetic pull

If the length of air gap is small, then even a small deflection or eccentricity of the shaft would produce a large irregularity in the length of air gap. It is responsible for production of large unbalanced magnetic pull which has the tendency to bend the shaft still more at a place where it is already bent resulting in fouling of rotor with stator. If the length of air gap of a machine is large, a small eccentricity would not be able to produce noticeable unbalanced magnetic pull.

Cooling

If the length of air gap is large, the cylindrical surfaces of rotor and stator are separated by a large distance. This would afford better facilities for cooling at the gap surfaces especially when a fan is fitted for circulation of air.

Noise

The principal cause of noise in induction motors is the variation of reluctance of the path of the zig zag leakage flux. To ensure that the noise produced will not be objectionable, it is necessary to make the zig zag leakage as small as possible. This can be done by increasing the length of the air-gap.

> From the above, we conclude that the length of air gap in an induction machine should be as small as mechanically possible in order to keep down the magnetizing current and to improve the power factor. This is a major consideration. But if a higher overload capacity, better cooling, reduction in noise or reduction in unbalanced magnetic pull is important, then large air gap lengths should be used.

RELATIONS FOR CALCULATION OF LENGTH OF AIR GAP

(i) In order to estimate the length of air gap of small induction motors, the following expression can be used

$$l_g = 0.2 + 2\sqrt{DL} \quad mm$$

where D and L are expressed in metre. The air gap is a mere clearance between rotor and stator and is made smaller than the value given by Eqn. if roller and ball bearings are used.

ii) The following relation may also be usefully used

$$l_g = 0.2 + D \text{ mm}$$

where D is expressed in metre.

(iv) For machines with journal bearings, following expression may be used

$$l_g = 1.6\sqrt{D} - 0.25 \text{ mm}$$

where D is expressed in metre.

Rotor diameter D_r = stator bore − 2 × length of air gap = $D - 2l_g$

CHOICE OF ROTOR SLOTS FOR SQUIRREL CAGE MACHINES

With certain combinations of stator and rotor slots, the following problems may develop in the induction motor.

- The motor may refuse to start.
- The motor may crawl at some subsynchronous speed.
- Severe vibrations are developed and so the noise will be excessive.

The above effects are due to harmonic magnetic fields developed in the machine. The harmonic fields are due to winding, slotting, saturation and irregularities in air gap.

The squirrel cage rotor will circulate currents due to any harmonic emf produced by the gap flux except that has a wavelength equal to the pitch of the bars. The effects of space harmonic fields produced by windings are greatly intensified by slotting. The slots introduces steps in the mmf wave and produces further harmonics and also modulates the gap flux. Hence the choice of rotor slots is particularly important in the case of squirrel cage machines. Any bad combination of stator and rotor slots may result in awkward behaviour.

NOTE:

1. CRAWING

If the mechanical load on the shaft requires a constant load torque and if the torque developed by the rotor is below this load torque then the motor cannot accelerate upto its full speed but continues to run at a speed little lower than 1/7th synchronous speed. This condition of the motor is called crawling.

2. COGGING

When the number of rotor slots is equal to the number of stator slots, the speeds of all the harmonics produced by stator slotting coincide with the speed of corresponding rotor harmonics. Thus harmonics of every order would try to exert synchronous torques at their corresponding synchronous speeds and the machine would refuse to start. This is known as cogging.

RULES FOR SELECTING ROTOR SLOTS OF SQUIRREL CAGE MACHINES

The following general rules should be followed concerning the choice of rotor slots for squirrel cage machines.

(i) As stated earlier, the number of rotor slots should never be equal to stator slots but must either be large or smaller. Satisfactory results are obtained when the number of rotor slots is 15 to 30 per cent larger or smaller than the number of stator slots.

(ii) The difference between stator slots and rotor slots should not be equal to p, $2p$ or $5p$ to avoid synchronous cusps.

(iii) The difference between the number of stator and rotor slots should not be equal to $3p$ for 3 phase machines in order to avoid magnetic locking.

(iv) The difference between number of stator slots and rotor slots should not be equal to, 1.2, $(p \pm 1)$ or $(p \pm 2)$ to avoid noise and vibrations.

Summarizing,

$$\boxed{\begin{array}{l} S_s - S_r \text{ should not be equal to} \\ \qquad 0, \pm p, \pm 2p, \pm 3p, \pm 5p \\ \qquad \pm 1, \pm 2, \pm (p \pm 1), \pm (p \pm 2). \end{array}}$$

DESIGN OF ROTOR BARS AND SLOTS

The squirrel cage rotor consists of a laminated core, rotor bars and end-rings. The rotor bars and end rings are made of aluminium or copper. The length of the rotor is same as that of stator. Some manufacturers, keep the length of rotor slightly higher than that of stator, in order to utilize the end fluxes. The diameter of the rotor is slightly lesser than the stator to avoid mechanical friction between the stationary stator and rotating rotor.

$$\text{The diameter of rotor, } D_r = D - 2l_g$$

where, D = Diameter of stator bore

l_g = Length of air gap

1. Design of rotor bars

For a 3-phase machine, the rotor bar current is given by the equation,

$$\text{Rotor bar current, } I_b = \frac{6I_s T_s}{S_r} K_{ws} \cos\phi$$

$$\approx 0.85 \frac{6I_s T_s}{S_r}$$

where, I_s = Stator current per phase
T_s = Stator turns per phase
S_r = Number of rotor slots

$$\text{Area of each rotor bar, } a_b = \frac{I_b}{\delta_b} \text{ in mm}^2$$

The current density in the rotor bar, δ_b may be taken between 4 to7A/mm².

$$\text{Length of bar} \quad L_b = L + 0.045 \quad \text{m}$$

$$\text{Total Copper loss in bar} = S\,I_b{}^2\,\frac{\rho L_b}{a_b}$$

In case of squirrel cage motor the cross-section of bars will take the shape of the slot and insulation is not used between bars and rotor core.

2. **Design ofslots**

The rotor slots for squirrel cage rotor may either be closed or semi-enclosed types

Fig.   Types of rotor slots.

The semi closed slots provides better overload capacity.

Advantages of closed slots

- Low reluctance
- Less magnetizing current
- Quieter operation
- Large leakage reactance and so starting current is limited

Disadvantage of closed slots

- Reduced over load capacity

Generally, the rotor slots and so the rotor bars are rectangular in shape. In rectangular bars, during starting most of current flows through top portion of the bar and so the effective rotor resistance is increased. This improves the starting torque.

DESIGN OF END RINGS

The distribution of current in the bars and end rings of a squirrel cage motor is complicated. It can be shown that if flux distribution is sinusoidal then the bar current and end ring current will also be sinusoidal.

End ring current $\quad I_e = \dfrac{S_r I_b}{\pi p}$

Area of cross section of end ring $a_e = \dfrac{I_e}{\delta_e}$

Let current density in the end ring δ_e be 4 to 7 A/mm^2

Also

Area of cross section of end ring = Depth of end ring x Thickness of end ring

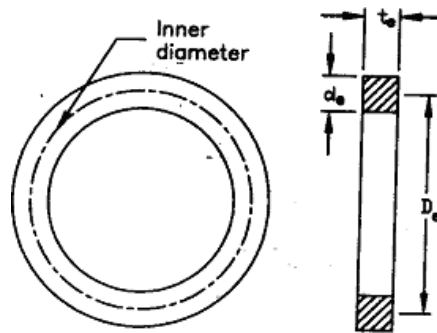$$a_e = d_e \times t_e$$

Fig. shows the dimensions of end ring.

Fig. Dimensions of end ring.

Total Copper loss in end ring $2I_e \, \rho \, \dfrac{\pi D_r}{a_e}$

<u>Note:</u>

1. $\dfrac{s}{1-s} = \dfrac{\text{Rotor cu loss}}{\text{output}}$

2. $N_s = \dfrac{120 f}{p}$

3. $N_r = (1-s)N_s$

<u>REDUCTION OF HARMONICS TORQUE</u>

     The methods used for reduction or elimination of harmonic torques are chording, integral slot winding, skewing and increasing the length of air gap.

     Chording : The chorded windings with integral number of slots per pole per phase weakens the stator winding mmf harmonics.

     Integral slot winding : Windings with fractional number of slots per pole per phase create asymmetrical mmf distribution around the air gap and favour the creation of noise in the motor. Therefore, fractional slot windings are not used for induction motor stator and only integral slot windings are used.

     Skewing : The motor noise, vibrations, cogging and synchronous cusps can be reduced or even entirely eliminated by skewing either the stator or the rotor. The practice generally followed in India is to skew the rotor.

     In order to eliminate the effect of any harmonic, the rotor bars should be skewed through an angle so that the bars lie under alternate harmonic poles of the same polarity or in other words the bars must be skewed through two pitches.

     $\therefore$ For elimination of nth harmonic by skewing,

     Angle of skew, $\theta = \dfrac{720}{np}$ deg. mech.

     Increasing air gap length : The increase in air gap length reduces the harmonic torques but increases the no load current and results in poor power factor. Hence only for mechanical reasons the air gap is made larger.

EXAMPLE: 01

Estimate the main dimensions, air-gap length, stator slots, stator turns per phase and cross sectional area of stator and rotor conductors for a 3-phase, 15HP, 400V, 6 pole, 50 Hz, 975 rpm, induction motor. The motor is suitable for star delta starting. $B_{av} = 0.45$ Wb/m², ac = 20000 amp.cond./m, $L/\tau = 0.85$, $\eta = 0.9$, pf = 0.85.

Given Data

3 - phase	400 V	$B_{av} = 0.45$ Wb/m²
15 HP	$L/\tau = 0.85$	ac = 20000 amp.cond./m.
50 Hz	6 pole	$\eta = 0.9$
pf = 0.85	975 rpm	star-delta starting

Solution

$$\text{kVA input}, Q = \frac{HP \times 0.746}{\eta \times pf} = \frac{15 \times 0.746}{0.9 \times 0.85} = 14.63 \text{ kVA}$$

$$\text{Output coefficient}, C_o = 11 B_{av} \text{ ac } K_{ws} \times 10^{-3}$$
$$= 11 \times 0.45 \times 20000 \times 0.955 \times 10^{-3}$$
$$= 94.545 \text{ kVA / m}^3 \text{ - rps}$$

$$\text{Synchronous speed}, n_s = \frac{2f}{p} = \frac{2 \times 50}{6} = 16.667 \text{ rps}$$

We know that, $Q = C_o D^2 L n_s$

$$\therefore D^2 L = \frac{Q}{C_o n_s} = \frac{14.63}{94.545 \times 16.667} = 9.284 \times 10^{-3}$$

$L/\tau = 0.85$	Put, $L = 0.445 D$ in the equation for D^2L
$\therefore L = 0.85\tau$	$\therefore D^2(0.445D) = 9.284 \times 10^{-3}$
$L = 0.85\frac{\pi D}{p}$	$D = \left(\frac{9.284 \times 10^{-3}}{0.445}\right)^{1/3} = 0.2753m \approx 0.275m$
$= \frac{0.85 \times \pi}{6} D = 0.445D$	$L = 0.445D = 0.445 \times 0.275 = 0.1224 \approx 0.12 \text{ m.}$

$$\phi_m = \frac{B_{av} \pi DL}{p} = \frac{0.45 \times \pi \times 0.275 \times 0.12}{6} = 7.775 \times 10^{-3} \text{ Wb}$$

For star delta starting, the motor should be designed for delta connection. In delta connection, phase voltage is equal to line voltage.

$$T_s = \frac{E_s}{4.44 \, f \, \phi_m K_{ws}} = \frac{400}{4.44 \times 50 \times 7.775 \times 10^{-3} \times 0.955} = 242.66 \approx 242$$

Total conductors = $6 T_s = 6 \times 242 = 1452$ conductors

Slot pitch lies between 15 mm to 25 mm.

When $y_{ss} = 15mm$	When $y_{ss} = 25mm$
$S_s = \dfrac{\pi D}{y_{ss}} = \dfrac{\pi \times 0.275}{15 \times 10^{-3}} = 58$	$S_s = \dfrac{\pi D}{y_{ss}} = \dfrac{\pi \times 0.275}{25 \times 10^{-3}} = 34.55 \approx 34$

The number of slots lies between 34 and 58.

The stator slots should be multiple of q, where q is slots per pole per phase.

Stator slots, S_s = Number of phases × poles × q

When q = 2, $S_s = 3 \times 6 \times 2 = 36$
When q = 3, $S_s = 3 \times 6 \times 3 = 54$
When q = 4, $S_s = 3 \times 6 \times 4 = 72$

Let $S_s = 36$

$$\therefore Z_{ss} = \frac{6T_s}{S_s} = \frac{1452}{36} = 40.33 \approx 40$$

Total stator conductors $= S_s \times Z_{ss} = 36 \times 40 = 1440$ conductors

New value of turns per phase, $T_s = \dfrac{Z_{ss} \times S_s}{6} = \dfrac{40 \times 36}{6} = 240$

kVA input, $Q = \sqrt{3}V_L \times I_L \times 10^{-3} = 3E_{ph}I_{ph}10^{-3}$

$$\therefore I_{ph} = \frac{Q \times 10^3}{3E_{ph}} = \frac{14.62 \times 10^3}{3 \times 400} = 12.183 \text{ A}$$

Let, $\delta = 3A / mm^2$

$$\therefore a_s = \frac{I_{ph}}{\delta} = \frac{12.183}{3} = 4.061 \text{ mm}^2$$

Area of cross-section of stator conductor, $a_s = 4.061$ mm²

Length of air gap, $l_g = 0.2 + 2\sqrt{DL} = 0.2 + 2\sqrt{0.275 \times 0.12} = 0.5633$mm

Let, $l_g = 0.6$mm

Rotor slots

Let, S_r = Number of rotor slots   ; S_s = Number of stator slots

We know that, $(S_s - S_r)$ cannot be, $0, \pm p, \pm 2p, \pm 3p, \pm 5p, \pm 1, \pm 2, \pm(p \pm 1), \pm(p \pm 2)$

Here, p = 6

$\therefore (S_s - S_r)$ cannot be, $0, \pm 6, \pm 12, \pm 18, \pm 30, \pm 1, \pm 2, \pm 5, \pm 7, \pm 8, \pm 4$

Here, $(S_s - S_r)$ can be, $\pm 3, \pm 9, \pm 10, \pm 11$, etc.,

Let, $S_s - S_r = \pm 3$ $\therefore S_r = S_s + 3$ or $S_r = S_s - 3$

Let, $S_r = S_s - 3 = 36 - 3 = 33$

Rotor bar current, $I_b = 0.85 \dfrac{6T_s I_s}{S_r} = \dfrac{0.85 \times 6 \times 240 \times 12.183}{33}$

$$= 451.88 \text{amps}$$

Let, $\delta_b = 4 A / mm^2$

$$\left.\begin{array}{l}\text{Area of cross section}\\\text{of rotor bar}\end{array}\right\} a_b = \frac{I_b}{\delta_b} = \frac{451.88}{4} = 112.96 = 113 \text{ mm}^2$$

End ring current, $I_e = \dfrac{S_r I_b}{\pi p} = \dfrac{33 \times 451.88}{\pi \times 6} = 791.1 \text{amps}$

Let, $\delta_e = 4 \text{ A/mm}^2$

Area of cross section of end ring, $a_e = \dfrac{I_e}{\delta_e} = \dfrac{791.1}{4} = 197.775 \text{ mm}^2$

Let, $a_e = 200 \text{ mm}^2$

Result

Diameter of stator	=	0.275 m
Length of stator	=	0.12 m
Turns per phase	=	240 turns
Number of stator slots	=	36
Number of rotor slots	=	33
Area of cross-section of stator conductor	=	4.061 mm²
Area of cross-section of rotor bar	=	113 mm²
Area of cross-section of end ring	=	200 mm²

EXAMPLE: 02

Design a cage rotor for a 40 HP, 3-phase, 400V, 50 Hz, 6 pole, delta connected induction motor having a full load η of 87% and a full load pf of 0.85. Take D = 33 cm and L = 17 cm. Stator slots = 54, conductors per slot = 14. Assume suitably the missing data if any.

Given Data

3 - phase	6 pole	$S_s = 54$
40 HP	delta connected	$Z_{ss} = 14$
400 V	η = 0.87	D = 33 cm
50 Hz	pf = 0.85	L = 17 cm

Solution

We know that, $(S_s - S_r)$ cannot be, $0, \pm p, \pm 2p, \pm 3p, \pm 5p,$
$$\pm 1, \pm 2, \pm(p \pm 1), \pm(p \pm 2)$$

Here p = 6, $\therefore (S_s - S_r)$ cannot be, $0, \pm 6, \pm 12, \pm 18, \pm 30,$
$$\pm 1, \pm 2, \pm 7, \pm 6, \pm 8, \pm 4.$$

$(S_s - S_r)$ can be ± 3 or ± 9

Let rotor slots, $S_r = S_s - 3 = 54 - 3 = 51$

Total stator conductors = $6 T_s$

$\therefore$ Turns per phase, $T_s = \dfrac{\text{Total stator conductors}}{6}$

$$= \dfrac{\text{Slots} \times \text{conductors per slot}}{6} = \dfrac{54 \times 14}{6} = 126$$

Input kVA, $Q = \dfrac{HP \times 0.746}{\eta \times pf} = \dfrac{40 \times 0.746}{0.87 \times 0.85} = 40.351 \text{ kVA}$

Also input kVA, $Q = 3\,E_{ph}\,I_{ph} \times 10^{-3}$

$$\therefore I_{ph} = I_s = \frac{Q}{3\,E_{ph} \times 10^{-3}} = \frac{40.351}{3 \times 400 \times 10^{-3}} = 33.62 \text{ amps}$$

(stator is delta connected, $\therefore E_{ph} = V_L = 400V$)

Rotor bar current, $I_b = \dfrac{6\,T_s\,I_s}{S_r} \times 0.85 = \dfrac{6 \times 126 \times 33.62}{51} \times 0.85 = 423.6 \text{ amps}$

Let $\delta_b = 4A / mm^2$

$$\therefore a_b = \frac{I_b}{\delta_b} = \frac{423.6}{4} = 105.9 = 106 \text{ mm}^2$$

End ring current, $I_e = \dfrac{S_r I_b}{\pi p} = \dfrac{51 \times 423.6}{\pi \times 6} = 1146.139 \text{amps}$

Let $\delta_e = 4A / mm^2$

$$a_e = \frac{I_e}{\delta_e} = \frac{1146.139}{4} = 286.53 \text{ mm}^2$$

In induction motors the length of rotor core is same as that of stator core.

$\therefore$ Length of rotor core, $L_r = 17 \text{ cm} = 0.17 \text{ m}$

Length of air gap, $l_g = 0.2 + 2\sqrt{DL} = 0.2 + 2\sqrt{0.33 \times 0.17}$

$$= 0.67 \text{ mm} = 0.7 \text{ mm}$$

Diameter of rotor, $D_r = D - 2\,l_g$

$$D_r = 0.33 - 2 \times 0.7 \times 10^{-3}$$

$$D_r = 0.3286 \text{ m}$$

Results

Length of rotor	=	0.17 m
Diameter of rotor	=	0.3286 m
Length of air-gap	=	0.7 mm

EXAMPLE: 03

A 11 kW, 3 phase, 6 pole, 50 Hz, 220 V, star connected induction motor has 54 stator slots, each containing 9 conductors. Calculate the values of bar and end ring currents. The number of rotor bars is 64. The machine has an efficiency of 0.86 and a power factor of 0.85. The rotor mmf may be assumed as 85 per cent of stator mmf.

Also find the bar and the end ring sections if the current density is 5 A/mm².

GIVEN DATA

P=11 kW   3 phase   p=6   f=50 Hz   V=220 V   stat connected S_s=54
Conductors/slot=9 S_r=64   η=0.86   cosΦ=0.85   δ=5 A/mm²
Rotor mmf=0.85 Stator mmf

Solution. Stator current per phase

$$I_s = \frac{11 \times 1000}{\sqrt{3} \times 220 \times 0.86 \times 0.85} = 40 \text{ A.}$$

Number of stator conductors $= 54 \times 9 = 486.$

∴   Stator turns/phase $T_s = 486/6 = 81.$

Stator mmf $= 3 \, I_s T_s = 3 \times 40 \times 81 = 9720 \text{ A.}$

∴   Rotor mmf $= 0.85 \times 9720 = 8250 \text{ A.}$

But rotor mmf $= S_r \, I_b/2 = 32 \, I_b$

∴ $32 \, I_b = 8250$ or   current in rotor bars $I_b = 258 \text{ A.}$

End ring current $I_s = \dfrac{S_r I_b}{\pi p} = \dfrac{64 \times 258}{\pi \times 6} = 883 \text{ A.}$

∴   Area of each bar $a_b = 258/5 = 51.6 \text{ mm}^2.$

and of each end ring $a_e = 883/5 = 176.6 \text{ mm}^2.$

EXAMPLE: 03

       A 3 phase 2 pole, 50 Hz squirrel cage induction motor has a rotor diameter 0.20 m and core length 0.12 m. The peak density in the air gap is 0.55 Wb/m². The rotor has 33 bars, each of resistance 125 μΩ and a leakage inductance 2 μH. The slip is 6%.

     Calculate (i) the peak value of current in each bar (ii) rotor I²R loss (iii) rotor output and (iv) torque exerted. Neglect the resistance of end rings.

GIVEN DATA

3 phase   p=2   f=50 Hz D_r=0.20 m   L=0.12m B_m=0.55 Wb/m² S_r=33

R/bar=125μΩ   L/bar=2 μH   S=0.06

Solution

Synchronous speed $n_s = \dfrac{2f}{2} = \dfrac{2 \times 50}{2} = 50$ ~~Hz~~ rps

Actual speed of rotor $= n_r = (1-s)\,n_s = (1-0.06) \times 50$

$$= 47 \text{ rps}$$

Peripheral speed of stator field $= \pi D n_s = \pi \times 0.2 \times 50$

$$= 31.42 \text{ m/s}$$

Peripheral speed of rotor $= \pi D n_r = \pi \times 0.2 \times 47$

$$= 29.53 \text{ m/s}$$

Relative speed of rotor bars with respect to stator field

$$\nu = 31.42 - 29.53$$
$$= 1.89 \text{ m/s}$$

Maximum e.m.f in each bar $= B_m \, L \, \nu$

$$= 0.55 \times 0.12 \times 1.89 = 0.125 \text{ V}$$

slip frequency $f' = sf = 0.06 \times 50 = 3 \text{ Hz}$

Reactance of each bar $= X_b = 2 \pi f' L$

$$= 2\pi \times 3 \times 2 = 37.7 \ \mu\Omega$$

Impedance of each bar $Z_b = \sqrt{r_b^2 + x_b^2}$

$$= \sqrt{(125)^2 + (37.7)^2} = 130.6 \ \mu\Omega$$

Maximum current in each bar $= I_{bmax} = \dfrac{E_{max}}{Z_b}$

$$= \dfrac{0.125}{130.6 \times 10^{-6}} = 957.1 \ A$$

R.ms value of current in each bar $I_b = \dfrac{I_{bmax}}{\sqrt{2}}$

$$I_b = \dfrac{957.1}{\sqrt{2}} = 676.8 \ A$$

Rotor I^2R loss $= S_r \ I_b^2 \ r_b = 33 \times 676.8^2 \times 125 \times 10^{-6}$

$$= 1890 \ W$$

output $=$ rotor I^2R loss $\times \dfrac{(1-S)}{S} = 1890 \times \dfrac{(1-0.06)}{0.06}$

$$= 29.61 \ kW$$

Torque $T = \dfrac{output}{2\pi n_r} = \dfrac{29.61 \times 10^3}{2\pi \times 47} = 100.3 \ Nm$

$I_{bmax} = 957.1 \ A$ Rotor I^2R loss $= 1890 \ W$

Rotor output $= 29.61 \ kW$ Torque $T = 100.3 \ Nm$

EXAMPLE: 04

following data :   *A 15 kW, 3 phase, 6 pole, 50 Hz squirrel cage induction motor has the*

     Stator bore diameter = 0.32 m ; axial length of stator core = 0.125 m ; number of stator slots = 54 ; number of conductors per stator slot = 24 ; current in each stator conductor = 17.5 A ; full load power factor = 0.85 lagging.

     Design a suitable cage rotor giving number of rotor slots, section of each bar and section of each ring. The full load speed is to be about 950 r.p.m. approximately. Use copper for the rotor bars and end rings. Resistivity of copper is 0.02 Ω/m and mm².

GIVEN DATA

P=15 kW   3 phase   p=6   f=50 Hz   D=0.32 m   L=0.125 m S_s=54
Conductors/stator slot=24 I_s=17.5 A   cosΦ=0.85   N=950 rpm   ρ=0.02Ω/m and mm²

Solution

we know that

$(S_s - S_r)$ cannot be

$0, \pm P, \pm 2P, \pm 3P, \pm 5P, \pm 1, \pm 2, \pm(P\pm 1), \pm(P\pm 2)$

Here $P = 6$

$\therefore$ $(S_s - S_r)$ cannot be

$\quad$ $0, \pm 6, \pm 12, \pm 18, \pm 30, \pm 1, \pm 2, \pm 5, \pm 7, \pm 8, \pm 4$

Here $(S_s - S_r)$ can be $\pm 3, \pm 9, \pm 10, \pm 11$ etc

Let $\quad S_s - S_r = 3 \qquad S_r = S_s - 3 = 54 - 3 = 51$

$$\boxed{S_r = 51}$$

Stator turns per phase $T_s = \dfrac{S_s \, Z_{ss}}{6} = \dfrac{54 \times 24}{6}$

$$T_s = 216$$

Rotor bar current $\quad I_b = \dfrac{0.85 \times 6 \, I_s T_s}{S_r}$

$$I_b = \dfrac{0.85 \times 6 \times 17.5 \times 216}{51} = 378 \text{ A}$$

$$I_b = 378 \text{ A}$$

Area of each bar $\quad a_b = \dfrac{I_b}{\delta_b} = \dfrac{378}{7} = 54 \text{ mm}^2$

$$a_b = 54 \text{ mm}^2$$

Allowing 45 mm for projection of bar beyond core and skewing, length of each bar

$$L_b = L + 0.045 = 0.125 + 0.045 = 0.17 \text{ m}$$

$$L_b = 0.17 \text{ m}$$

Copper loss in bars $= S_r \, I_b^2 \, \dfrac{\rho L_b}{a_b} = \dfrac{51 \times 378^2 \times 0.02 \times 0.17}{54}$

$$= 458.81 \, \omega$$

current in each end ring $\quad I_e = \dfrac{S_r I_b}{\pi p} = \dfrac{51 \times 378}{\pi \times 6}$

$$I_e = 1022.72 \text{ A}$$

Area of end ring $\quad a_e = \dfrac{I_e}{\delta_e} = \dfrac{1022.72}{7} = 146.10 \text{ mm}^2$

copper loss in end ring $= 2 I_e^2 \, \dfrac{\rho \pi D_r}{a_e}$

$$= \frac{2 \times 1022.72 \times 0.02 \times \pi \times 0.32}{146.10} = 287.88 \, \omega$$

Total loss = cu loss in bars + cu loss in end ring

Total loss = $458.81 + 287.88 = 746.69 \, \omega$

CHECK: $\dfrac{S}{1-S} = \dfrac{\text{Rotor cu loss}}{\text{output}}$; $\dfrac{S}{1-S} = \dfrac{746.69}{15000} = 0.0497$

$\therefore \quad S = 0.0473$

$n_s = \dfrac{2f}{P} = \dfrac{2 \times 50}{6} = 16.66 \, rps$

$n_r = (1-S) \, n_s = (1 - 0.0473) \, 16.66 = 15.87 \, rps$; $N_r = n_s \times 60$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad = 15.87 \times 60$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad N_r = 952.2 \, rpm$

This is nearly equal to specified speed (i.e. 950 rpm)

DESIGN OF SLIP RING ROTOR

The wound rotor has the facility of adding external resistance to rotor circuit in order to improve the torque developed by the motor. The rotor consists of laminated core with semi-enclosed slots and carries a three phase winding.

1. Rotor Windings

For small induction motors of slip ring type, it is a normal practice to use mush windings for rotor housed in semi-enclosed slots. The coils are roughly formed outside the machine and dropped into the slots through slot opening one by one. It is usual to use several wires in parallel per turn, to keep the conductor small enough to go through the narrow slot opening. The rotor is invariable star connected and three leads are brought through the shaft to the slip rings.

For larger motors, a double layer bar type wave winding is used. This winding has generally two bars per slot. The bars are pushed through partially closed slots and are bent to shape at the other end (See Fig. 7.62). In motors of outputs of about 750 kW and over, we have to use 4 bars sometimes. The use of 4 bars per slot is made to reduce the current handled by each slip ring. The winding with more than 2 bars per slot is called a barrel winding and is usually wave wound.

2. Number of Rotor Turns

The rotor is equivalent to secondary of a transformer and the voltage between slip rings is maximum when the rotor is at rest. The rotor voltage on open circuit between slip rings should not exceed 500 volt for small machines where hand operated starters and switchgear are employed. For large size machines the voltage between slip rings can be upto 2000 Volt.

Let T_s, T_r = number of turns per phase for stator and rotor respectively,

   K_{ws}, K_{wr} = winding factor for stator and rotor respectively,

   E_s = stator voltage per phase,

   E_r = rotor voltage per phase at standstill.

Now $\dfrac{E_r}{E_s} = \dfrac{K_{wr}\,T_r}{K_{ws}\,T_s}$

∴ Rotor turns per phase $T_r = \dfrac{K_{ws}}{K_{wr}} \cdot \dfrac{E_r}{E_s} \cdot T_s$ ...(10.19)

In case of small machines :

E_r should not exceed 500 V and $500/\sqrt{3} = 290$ V for delta and star connected machines respectively.

By assuming a suitable value of voltage between slip rings, the rotor turns per phase to be provided can be calculated from Eqn. 10.19.

3. Rotor Current and Area of Rotor Conductor

The full load rotor mmf is taken as 85 per cent of stator mmf.

∴ $I_r T_r = 0.85\, I_s T_s$ or $I_r = 0.85\, \dfrac{I_s T_s}{T_r}$

where I_r = rotor current per phase.

The area of the rotor conductors is found out by assuming a suitable value for current density. In order to avoid excessive rotor copper loss, the current density in the rotor is chosen almost equal to that in the stator.

The range of current density in rotor is 3 to 5 A/mm²

Let, δ_r = Current density in rotor

∴ Area of rotor conductor, $a_r = \dfrac{I_r}{\delta_r}$

4. Number of Rotor Slots

The discussions made on choice of squirrel cage rotor slots are also applicable to the choice of wound rotor slots. For wound rotors the windings are always three phase winding and they are star connected at one end and the other three end are terminated on three slip rings mounted on the shaft.

Since the windings are three phase windings, the number of slots should be such that a balanced winding is obtained. Generally windings with an integral number of slots per pole per phase are used for the rotor. When fractional slot windings are used, it is preferable to have the number of slots as multiples of phases and pair of poles.

NOTE:

1. DISPERSION COEFFICIENT

The dispersion coefficient is defined as the ratio of magnetizing current to ideal short circuit current.

Dispersion coefficient, $\sigma = I_m / I_{sci}$

where, I_m = Magnetizing current

I_{sci} = E_s/X_s = Ideal short circuit current

E_s = Stator phase voltage

X_s = Total leakage reactance of the motor referred to stator

Higher value of dispersion coefficient results in

- Poor power factor
- Reduced over-load capacity
- Reduced output

2. FULL LOAD SLIP

The value of slip at full load is determined by the rotor resistance. A reasonable value of rotor resistance to be incorporated in the rotor can be obtained by the knowledge of reasonable values of full load slip. The value of slip, s, is derived from the following relationship

$$\frac{\text{rotor copper loss}}{\text{rotor output}} = \frac{s}{1-s}$$

where s is the per unit slip.

3. Coil Span = $\dfrac{\text{Rotor slots}}{\text{Number of poles}}$

EXAMPLE: 01

A 3-phase induction motor has 54 stator slots with 8 conductors per slot and 72 rotor slots with 4 conductors per slot. Find the number of stator and rotor turns. Find the voltage across the rotor slip rings, when the rotor is open circuited and at rest. Both stator and rotor are star connected and a voltage of 400 volt is applied across the stator terminals.

Given Data

3-phase	Stator conductors per slot = 8
$S_s = 54$	Rotor conductors per slot = 4.
$S_r = 72$	

Solution

Stator conductor per slot, $Z_{ss} = \dfrac{6T_s}{S_s}$

$\therefore$ Stator turns per phase, $T_s = \dfrac{Z_{ss} \times S_s}{6} = \dfrac{8 \times 54}{6} = 72$

Rotor conductors per slot, $Z_{rs} = \dfrac{6T_r}{S_r}$

$\therefore$ Rotor turns per phase, $T_r = \dfrac{Z_{rs} \times S_r}{6} = \dfrac{4 \times 72}{6} = 48$

Let us assume that, $K_{ws} = K_{wr}$

$\therefore$ The turns ratio of induction motor can be written as

$$\frac{E_r}{E_s} = \frac{K_{wr}T_r}{K_{ws}T_s} = \frac{T_r}{T_s}$$

$\therefore$ Rotor emf at standstill, $E_r = E_s \dfrac{T_r}{T_s} = \dfrac{400}{\sqrt{3}} \times \dfrac{48}{72} = 153.96V \approx 154V$

Rotor emf between slip rings(line value) $= \sqrt{3}\, E_r = \sqrt{3} \times 154 = 266.7$ Volt

Result

Stator turns per phase	=	72
Rotor turns per phase	=	48
Rotor emf between slip rings at standstill	=	266.7 Volts

EXAMPLE: 02

A 90 kW, 500 V, 50 Hz, 3-phase, 8-pole induction motor has a star connected stator winding accommodated in 63 slots with 6 conductors per slot. If the slip ring voltage on open circuit is not to exceed 400 volt, find a suitable rotor winding by estimating number of slots, number of conductors per slot, coil span, slip-ring voltage on open circuit, approximate full load current per phase in rotor. Assume $\eta = 0.9$ and pf = 0.86.

Given Data

90 kW	500 V	Star connected stator
50 Hz	$S_s = 63$	Voltage between sliprings $\leq 400V$
3 - phase	$Z_{ss} = 6$	pf = 0.86
8-pole	$\eta = 0.9$	

Solution

Let rotor slots per pole per phase = q_r

Rotor slots should be multiple of q_r for integral slot winding. Number of rotor poles = 8 (same as stator poles).

Rotor slots, S_r = Number of phases × poles × q

For, $q_r = 2$, $S_r = 3 \times 8 \times 2 = 48$
For, $q_r = 3$, $S_r = 3 \times 8 \times 3 = 72$
For, $q_r = 4$, $S_r = 3 \times 8 \times 4 = 96$

To eliminate harmonics, $(S_s - S_r)$ should not be equal to

$$0, \pm p, \pm 2p, \pm 3p, \pm 5p, \pm 1, \pm 2,$$
$$\pm(p \pm 1), \pm(p \pm 2).$$

For p = 8, $(S_s - S_r)$ should not be equal to

$$0, \pm 8, \pm 16, \pm 24, \pm 40, \pm 1, \pm 2, \pm 9, \pm 7, \pm 10, \pm 6.$$

Here, $(S_s - S_r)$ can be equal to, $\pm 3, \pm 4, \pm 5, \pm 11, \pm 12, \pm 13, \pm 14, \pm 15$, etc.,

For $S_r = 48$, $S_s - S_r = 63 - 48 = 15$, hence the rotor slots can be 48, which also results in integral slot winding.

Let voltage between slip rings = 400 V

Rotor emf per phase, $E_r = 400 / \sqrt{3} = 231$ V

Stator emf per phase, $E_s = 500 / \sqrt{3} = 289$ V.

Stator turns per phase, $T_s = \dfrac{S_s Z_{ss}}{6} = \dfrac{63 \times 6}{6} = 63$

The turns ratio for induction motor is given by, $\dfrac{K_{wr}}{K_{ws}} \dfrac{T_r}{T_s} = \dfrac{E_r}{E_s}$

Let $K_{wr} = K_{ws}$

$\therefore$ Rotor turns per phase, $T_r = T_s \dfrac{E_r}{E_s}$

$\therefore T_r = 63 \times \dfrac{231}{289} \approx 50$

Rotor conductors per slot $= \dfrac{6T_r}{S_r} = \dfrac{6 \times 50}{48} = 6.25$

Let rotor conductors per slot = 6

New value of rotor turns per phase, $T_r = \dfrac{S_r Z_{sr}}{6} = \dfrac{48 \times 6}{6} = 48$

New value of rotor emf per phase, $E_r = E_s \times \dfrac{T_r}{T_s} = 289 \times \dfrac{48}{63} = 220V$

Emf between slip rings $= \sqrt{3} \ E_r = \sqrt{3} \times 220 = 381 V$

Rotor coil span for full pitch coils $= \dfrac{\text{Slots}}{\text{Number of poles}} = \dfrac{48}{8} = 6$ slots

kVA rating, $Q = \dfrac{kW}{\eta \times pf} = \dfrac{90}{0.9 \times 0.86} = 116.27$ kVA

Stator current per phase, $I_s = \dfrac{kVA}{3E_s \times 10^{-3}} = \dfrac{116.28}{3 \times 289 \times 10^{-3}} = 134.12 A$

Rotor current per phase, $I_r = \dfrac{0.85 \ I_s \ T_s}{T_r} = \dfrac{0.85 \times 134.12 \times 63}{48} = 149.63$ A

Let current density in rotor, $\delta_r = 5$ A/mm².

$\left. \begin{array}{l} \therefore \text{ Area of cross-section} \\ \text{of rotor conductors} \end{array} \right\}$ $a_r = \dfrac{I_r}{\delta_r} = \dfrac{149.63}{5} = 29.926 = 30 \text{ mm}^2$

Result

Number of stator slots	=	63
Number of rotor slots	=	48
Emf between slip rings	=	381 Volts
Rotor turns per phase	=	48
Rotor conductor per slot	=	6
Rotor current per phase	=	149.63 A
Cross-section of rotor conductor	=	30 mm²

LEAKAGE REACTANCE CALCULATION FOR POLYPHASE MACHINES (INDUCTION MACHINES)

Let Z_s = conductors per slot, q = slots per pole per phase :

p = number of poles, T_{ph} = turns per phase

Let us first consider only the slot leakage flux and the leakage reactance produced due to it. Fig.   (a) shows an arrangement where there are 2 slots of each phase under one pole. The width of each slot is W_s, the depth of the conductor portion is h and each slot contains Z_s conductors, there being $2Z_s$ conductors of each phase under one pole.
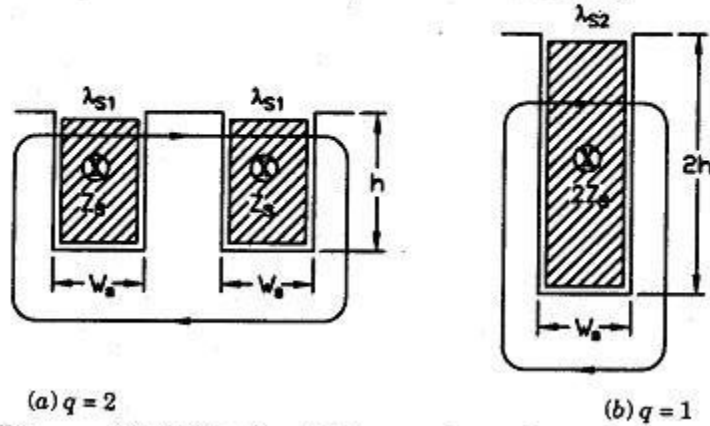

(a) q = 2   (b) q = 1

Fig.   Effect of number of slots per pole per phase on leakage flux.

The leakage reactance is directly proportional to specific slot leakage permeance.

Fig.   shows the slots of a phase under one pole. There are q slots per phase under one pole and they carry the same current. The conductors in slots of a phase under a pole produce leakage flux and this leakage flux has to traverse the slots of a phase under one pole (q slots).

Thus if λ_s is the specific slot permeance, the effective specific slot permeance for slots of a phase under one pole is λ_s/q, as these permeances are in series as shown in Fig. 3.59 (a). The conductors producing this leakage flux are :

$$T = \text{conductors per slot} \times \text{slots per pole per phase} = Z_s q.$$

reactance $X = 2\pi f\, T^2\, L\lambda$

Where L = Length of slot

Now for conductors of a phase under one pole $T = Z_s\, q$, also $\lambda = \lambda_s/q$.

∴ Slot leakage reactance of conductors of a phase under one pole

$$= 2\pi f(Z_s q)^2\, L\, \frac{\lambda_s}{q}\ = 2\pi f\, Z_s^2\, qL\lambda_s.$$

If the conductors of a phase under all the poles are connected in series, the slot leakage reactance per phase,

$$x_s = p \times 2\pi f Z_s^2\, qL\lambda_s = 2\pi f p q Z_s^2\, L\lambda_s$$

But $$Z_s = \frac{2T_{ph}}{qp}$$

$$x_s = 2\pi f p q \left(\frac{2T_{ph}}{qp}\right)^2 L\lambda_s$$

$$x_s = 8\pi f T_{ph}^2 L\left(\frac{\lambda_s}{qp}\right)$$

Similarly,

Overhang leakage reactance per phase, $x_0 = 8\pi f T_{ph}^2 L_0\left(\frac{\lambda_0}{qp}\right)$

Zigzag leakage reactances per phase $x_z = 8\pi f T_{ph}^2 L\left(\frac{\lambda_z}{qp}\right)$

Tooth top leakage reactance per phase $x_t = 8\pi f T_{ph}^2 L\left(\frac{\lambda_t}{qp}\right)$

Harmonic leakage reactance per phase $x_h = 8\pi f T_{ph}^2 L\left(\frac{\lambda_h}{qp}\right)$

EXAMPLE: 01

A 3 phase, 50 Hz, 6 pole induction motor has 3 slots per pole per phase. The stator core length is 0.12 cm and there are 225 turns per phase in stator. Two alternative sizes of almost equal area [Figs. (a) and (b)] are available for stator slots. Calculate the stator slot leakage reactance per phase in each case and comment on the result. The machine has a single layer winding.

Solution. For slot of Fig. (a).
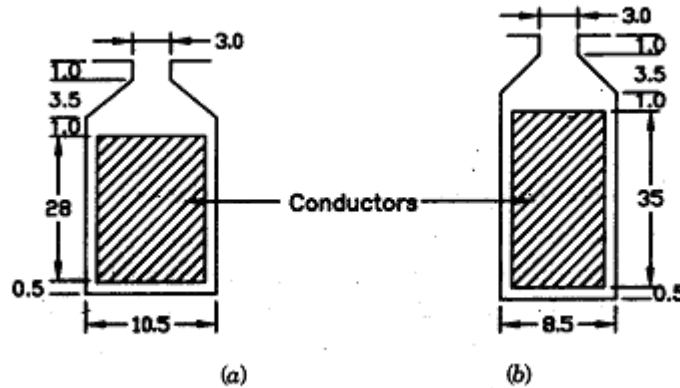
Fig. Stator slots (All dimensions in mm)

Specific slot permeance,

$$\lambda_s = \mu_0\left[\frac{h_1}{3W_s} + \frac{h_2}{W_s} + \frac{2h_3}{W_s + W_0} + \frac{h_4}{W_0}\right]$$

$$= 4\pi \times 10^{-7}\left[\frac{28}{3 \times 10.5} + \frac{1}{10.5} + \frac{2 \times 3.5}{10.5 + 3} + \frac{1}{3}\right] = 23.1 \times 10^{-7}$$

stator slot leakage reactance per phase

$$x_s = 8\pi f T_{ph}^2 L\left(\frac{\lambda_s}{pq}\right) = 8\pi \times 50 \times (225)^2 \times 0.12 \times \frac{23.1 \times 10^{-7}}{6 \times 3} = 0.98 \ \Omega.$$

From Fig. (b):

$$\lambda_s = 4\pi \times 10^{-7}\left[\frac{35}{3 \times 8.5} + \frac{1.0}{8.5} + \frac{2 \times 3.5}{8.5 + 3} + \frac{1}{3}\right] = 30.5 \times 10^{-7}$$

∴ Stator slot leakage reactance per phase

$$x_s = 8\pi \times 50 \times (225^2)^2 \times 0.12 \left(\frac{30.5 \times 10^{-7}}{6 \times 3} \right) = 1.29 \; \Omega.$$

The leakage reactance in the case of (b) is higher. Thus a deep and narrow slot gives a high value of leakage reactance·as compared with a shallow and wide slot.

EXAMPLE: 02

A 3-phase, 50 Hz alternator has parallel sided slots 0.6 m long, 20 mm wide and 100 mm deep. The conductors occupy a space 60 mm deep and the slot is closed by a non-magnetic wedge 20 mm deep. There are 8 poles with 4 slots per phase and 10 conductors per slot. The coils are full pitched and are connected in series. Determine the leakage reactance per phase due to the slot flux.

Solution

Height of insulation = Depth of slot − Height of wedge
                                     − Height of conductors

$$= 100 - 20 - 60 = 20 \; mm$$

The insulation is 10 mm at the top and 10 mm at the bottom of conductors with the insulation between conductors being neglected.

∴ Thickness of insulation is 10 mm.   i.e. $h_2 = 10 \; mm$

It is Given that h_1 = height of conductor = 60 mm
        and h_3 = height of wedge   = 20 mm

width of slot = width of opening   i.e. $w_s = w_o = 20 \; mm$

∴ specific permeance of slot

$$\lambda_s = \mu_o \left(\frac{h_1}{3 w_s} + \frac{h_2}{w_s} + \frac{2 h_3}{w_s + w_o} \right)$$

$$= 4\pi \times 10^{-7} \left(\frac{60}{3 \times 20} + \frac{10}{20} + \frac{2 \times 20}{20 + 20} \right) = 10\pi \times 10^{-7}$$

Total no. of slots = 3 pq = 3 × 8 × 4 = 96

Total conductors = No. of slots × conductors/slot

$$= 96 \times 10 = 960$$

$$T_{ph} = \frac{Z}{6} = \frac{960}{6} = 160$$

Leakage reactance due to slot leakage

$$x_s = 8\pi f \, T_{ph}^2 \, L \left(\frac{\lambda_s}{pq} \right)$$

$$= 8\pi \times 50 \times 160^2 \times 0.6 \left(\frac{10\pi \times 10^{-7}}{8 \times 4} \right) = 1.89 \; \Omega$$

$$\boxed{x_s = 1.89 \; \Omega}$$

OPERATING CHARACTERISTICS

1. MAGNETIZINGCURRENT

The no load current I_0 of an induction motor is made up of two components :
(i) Magnetizing current I_m, and (ii) Loss component of current I_l.
The magnetizing current is 90° out of phase with the voltage while the loss component is in phase with the voltage.

The magnetic circuit of a four pole induction motor is shown in Fig.   The flux produced by stator mmf turns passes through the following parts :

(i) air gap, (ii) rotor teeth, (iii) rotor core, (iv) stator teeth and (v) stator core.

The calculation of magnetizing current of an induction motor follows the same general procedure as the calculation of magnetizing current of a d.c. machine. The main difference is that whereas in a d.c. machine the flux is assumed to be uniform over any cross-section and the same mmf for all paths, in an induction motor the flux is distributed approximately sinusoidally and the mmf varies similarly.
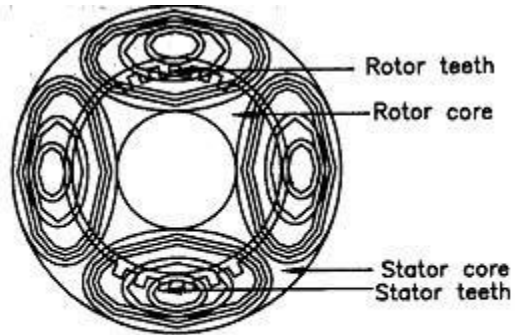
Fig. Magnetic circuit of a 4 pole induction motor.

If the value of flux density is calculated for the mean mmf, and a sinusoidal distribution of flux is assumed, the total flux obtained will be larger than true value ; or conversely the calculated magnetizing current for a given sinusoidal flux will be smaller than the true value.

If maximum values are taken instead the opposite result is obtained, i.e. flux is too small, or magnetizing current is too large.

Some intermediate position therefore will give a correct value. Though this position may differ somewhat in different motors, a flux tube crossing the air gap at 60° from the interpolar axis will always give a good approximation.

The reason for this is that the flux density distribution curve can be approximated too closely by a sine-wave with a third harmonic .   The value of flux density at 60° from the interpolar axis is the same whether the third harmonic is present or not. Thus the calculation of magnetizing mmf should be based upon the value of flux density at 60° from the interpolar axis as far as the gap and teeth are concerned.

$$\text{Mmf for air gap } AT_g = 800{,}000 \, B_{g60} \, K_g \, l_g$$

$$B_{g60} = 1.36 \, B_{av}$$

Total magnetizing mmf per pole for B_{60}

$AT_{60} = $ mmf for air gap+mmf for stator teeth+mmf for rotor teeth+ mf for stator core+mmf for rotor core

$$AT_{60} = AT_g + AT_{ts} + AT_{tr} + AT_{cs} + AT_{cr}$$

$$\text{Magnetizing current per phase } I_m = \frac{0.427 \, p \, AT_{60}}{K_{ws} \, T_s}$$

Note:

1. Distribution factor $K_d = \dfrac{\sin\left[\dfrac{m\beta}{2}\right]}{m\sin\left[\dfrac{\beta}{2}\right]}$

Where

$$m = \frac{slots}{poles \times phase}$$

$$\beta = \frac{180^o}{n}$$

$$n = \frac{slots}{pole}$$

2. Pitch factor $K_P = \cos\dfrac{\alpha}{2}$

3. Stator winding factor $K_{WS} = K_d K_P$

4. Area per pole $A = \dfrac{\pi DL}{p}$

5. Average flux density $B_{av} = \dfrac{\phi_m}{A}$

EXAMPLE: 01

A 75 kW, 3300 V, 50 Hz, 8 pole, 3 phase star connected induction motor has a magnetizing current which is 35 percent of the full load current. Calculate the value of stator turns per phase if the mmf required for flux density at 30° from pole axis is 500 A.

Assuming winding factor = 0.95, and full load efficiency and power factor 0.94 and 0.86 respectively.

Given data

$P = 75$ kW $E_L = 3300$V $f = 50$ Hz $p = 8$ $I_m = 0.35$ of full load current $AT_{60} = 500$ A
$K_{ws} = 0.95$ $\eta = 0.94$ $\cos\Phi = 0.86$

Solution. Full load current $= \dfrac{75 \times 1000}{\sqrt{3} \times 3000 \times 0.94 \times 0.86} = 17.9$ A.

∴   Magnetizing current $I_m = 0.35 \times 17.9 = 6.26$ A.

From Eqn. 10.27, magnetizing current $I_m = \dfrac{0.427\ pAT_{60}}{K_{ws}\ T_s}$

or stator turns per phase, $T_s = \dfrac{0.427\ p\ AT_{60}}{K_{ws}\ I_m}$

$= \dfrac{0.427 \times 8 \times 500}{0.95 \times 6.26} = 288.$

EXAMPLE: 02

A 15 kW, 400 V, 3 phase, 50 Hz, 6 pole induction motor has a diameter of 0.3 m and the length of core 0.12 m. The number of stator slots is 72 with 20 conductors per slot. The stator is delta connected. Calculate the value of magnetizing current per phase if the length of air gap is 0.55 m. The gap contraction factor is 1.2. Assume the mmf required for the iron parts to be 35 per cent of the air gap mmf. Coil span = 11 slots.

Given data

$P = 15$ kW $E_L = 400$ V $f = 50$ Hz $p = 6$ $D = 0.3$ m $L = 0.12$ m $S_S = 72$ $Z_{SS} = 20$ $l_g = 0.55$mm
$K_g = 1.2$ mmf required for iron path = 0.35 x air gap mmf   coil span = 11 slots

Solution

Distribution factor $K_d = \dfrac{\sin\left(\dfrac{m\beta}{2}\right)}{m \sin\left(\dfrac{\beta}{2}\right)}$

$m = \dfrac{\text{slots}}{\text{poles} \times \text{phases}} = \dfrac{72}{3 \times 6} = 4$

$\beta = \dfrac{180°}{n}$ $n = \dfrac{\text{slots}}{\text{poles}} = \dfrac{72}{6} = 12$

$\beta = \dfrac{180}{12} = 15°$

∴ $K_d = \dfrac{\sin\left(\dfrac{4 \times 15}{2}\right)}{4 \sin\left(\dfrac{15}{2}\right)} = 0.958$

slots/pole $= \frac{72}{6} = 12$ and coil span $= 11$ slots

∴ winding is chorded by 1 slot pitch

Angle of chording $\alpha = \frac{1}{12} \times 180 = 15°$

∴ Pitch factor $= \cos\frac{\alpha}{2} = \cos\frac{15}{2} = 0.9914$

$K_{ws} = K_d K_p = 0.958 \times 0.9914 = 0.95$

$T_s = \frac{Z_{ss} S_s}{6} = \frac{72 \times 20}{6} = 240$

$E = 400$ (stator is delta connected)

$\phi_m = \frac{E}{4.44 f\, T_{ph}\, K_{ws}} = \frac{400}{4.44 \times 50 \times 240 \times 0.95} = 7.9 \times 10^{-3}$ wb

Area/pole $= \frac{\pi DL}{P} = \frac{\pi \times 0.3 \times 0.12}{6} = 18.85 \times 10^{-3}$ m^2

$B_{av} = \frac{\phi_m}{A} = \frac{7.9 \times 10^{-3}}{18.85 \times 10^{-3}} = 0.418$ wb/m^2

$B_{g_{60}} = 1.36\, B_{av} = 1.36 \times 0.418 = 0.57$ wb/m^2

mmf required for air-gap $= 800000\, B_g K_g l_g$

$= 800000 \times 0.57 \times 1.2 \times 0.55 \times 10^{-3}$

$= 301$ A

mmf for iron parts $= 0.35 \times 301 = 105$ A

Total mmf $AT_{60} = $ mmf required for air gap + mmf for iron parts

$= 301 + 105 = 406$ A

Magnetizing current $I_m = \frac{0.427 \times P \times AT_{60}}{K_{ws} \times T_s} = \frac{0.427 \times 6 \times 406}{0.95 \times 240}$

$$\boxed{I_m = 4.56 \text{ A}}$$

SHORT CIRCUIT CURRENT

In order to find the value of short circuit (blocked rotor) current, the values of resistance and leakage reactance of the windings have to be evaluated.

Stator resistance

The stator resistance per phase

$$r_s = \rho\, T_s\, L_{mts}/a_s$$

where L_{mts} = length of mean turn of stator, m ;

and a_s = area of stator conductor, mm^2 .

The value of resistivity for copper is 0.021 Ω/m and mm^2 at 75°C.

Rotor Resistance

Wound Rotor.

The resistance of a wound rotor machine is found in a similar manner.

Rotor resistance per phase $r_r = \rho \dfrac{T_r \, L_{mtr}}{a_n}$

The rotor resistance per phase referred to stator, $r'_r = \left(\dfrac{K_{ws} \, T_s}{K_{wr} \, T_r}\right)^2 r_r$

Cage rotor

Let ρ = resistivity of material of bars and rings, Ω/m and mm^2 ;

L_b = length of each bar, m ; a_b = area of each bar, mm^2 ;

D_e = mean diameter of each ring, m ; and a_e = area of each ring, mm^2.

Resistance of each bar $\qquad r_b = \dfrac{\rho L_b}{a_b}$. Total copper loss in bars = $S_r \, I_b^2 \, \rho \, \dfrac{L_b}{a_b}$.

Resistance of each end ring $\qquad r_e = \rho \, \dfrac{\pi D_e}{a_e}$

Copper loss in two end rings $\qquad = 2 \, I_e^2 \, \dfrac{\rho \, \pi D_e}{a_e}$

$$= 2 \times \left(\dfrac{S_r \, I_b}{\pi p}\right)^2 \times \dfrac{\rho \pi D_e}{a_e} = \dfrac{2}{\pi} \, \rho \, \dfrac{I_b^2 \, S_r^2 \, D_e}{p^2 \, a_e} .$$

Total copper loss in rotor

$$= S_r \, I_b^2 \, \rho \, \dfrac{L_b}{a_b} + \dfrac{\pi}{2} \, \rho \, \dfrac{I_b^2 \, S_r^2 \, D_e}{p^2 \, a_e}$$

$$= \rho I_b^2 \, S_r^2 \left[\dfrac{L_b}{S_r \, a_b} + \dfrac{2}{\pi} \dfrac{D_e}{p^2 \, a_e}\right]$$

Hence total rotor resistance

$$= \rho \, S_r^2 \left(\dfrac{L_b}{S_r \, a_b} + \dfrac{2}{\pi} \dfrac{D_e}{p^2 \, a_e}\right)$$

The rotor resistance must be referred to stator in order to find out total resistance of motor as viewed from stator.

Let m_s, m_r = number of phases of stator and rotor respectively ;

T_s, T_r = number of turns of stator and rotor respectively ;

K_{ws}, K_{wr} = winding factor for stator and rotor respectively.

Total resistance of rotor referred to stator

$$= \left(\dfrac{m_s \, T_s \, K_{ws}}{m_r \, T_r \, K_{wr}}\right)^2 \times \rho \, S_r^2 \left(\dfrac{L_b}{S_r \, a_b} + \dfrac{2}{\pi} \dfrac{D_e}{p^2 \, a_e}\right)$$

The number of phases in a squirrel cage winding is equal to number of bars per pole or $m_r = S_r/p$.

The number of turns in series per phase for rotor is equal to the number of pole pairs or $T_r = p/2$ and rotor winding factor $K_{wr} = 1$.

Substituting the values for m_r, T_r and K_{wr}

Total resistance of cage rotor referred to stator

$$= \left[\frac{m_s \, T_s \, K_{ws}}{(S_r \,/\, p) \times (p\,/\,2) \times 1} \right]^2 \rho \, S_r^2 \left[\frac{L_b}{S_r \, a_b} + \frac{2}{\pi} \frac{D_e}{p^2 \, a_e} \right]$$

$$= 4 \, m_s^2 \, T_s^2 \, K_{ws}^2 \, \rho \left[\frac{L_b}{S \, a_b} + \frac{2}{\pi} \frac{D_e}{p^2 \, a_e} \right]$$

∴ Resistance of cage rotor, referred to stator, per phase

$$r'_r = 4 \, m_s \, T_s^2 \, K_{ws}^2 \, \rho \left[\frac{L_b}{S_r \, a_b} + \frac{2}{\pi} \frac{D_e}{p^2 \, a_e} \right]$$

FORMULAE USED FOR CALCULATION

1   $$r'_r = 4 \, m_s \, T_s^2 \, K_{ws}^2 \, \rho \left[\frac{L_b}{S_r \, a_b} + \frac{2}{\pi} \frac{D_e}{p^2 \, a_e} \right]$$

2   $$r'_r = \frac{\text{Total rotor culoss}}{m \, I'}$$

3   $$I_r = I_s \cos\phi_{sr}$$

4   $$I_b = \frac{2m_s T_s K_{ws} I'_r}{S_r}$$

5   $$I_e = \frac{S_r I_b}{\pi p}$$

EXAMPLE: 01

Calculate the equivalent resistance of rotor per phase referred to stator,
from the following data of a 400 V, 3 phase, 4 pole, 50 Hz cage motor.

Stator slots = 48 with 30 conductors per slot ; Rotor slots = 33 with one bar in each slot.
The length of each rotor bar is 0.12 m and area 60 mm². The end rings have a mean diameter of
0.18 m and an area of cross-section 150 mm².

Full pitch winding with 60° phase spread is used for the stator.

The material used for bars and end rings has a resistivity of 0.021 Ω/m and mm².

Given Data

$E_L = 400V$ $P = 4$ $f = 50 Hz$ $S_S = 48$ $Z_{ss} = 30$ $S_r = 33$

$L_b = 0.12 m$ $a_b = 60 mm^2$ $D_e = 0.18 m$ $a_e = 150 mm^2$

Full pitch winding   ∴ $K_p = 1$ $m\beta = 60°$ $\rho = 0.021 \, \Omega/m$ and mm^2

solution

$$m = \frac{\text{slots}}{\text{poles} \times \text{phases}} = \frac{48}{4 \times 3} = 4$$

$$K_d = \frac{\sin\left(\frac{m\beta}{2}\right)}{m \sin\left(\frac{\beta}{2}\right)} = \frac{\sin\frac{60}{2}}{4 \sin\left(\frac{60}{2 \times 4}\right)} = 0.958$$

$$K_{ws} = K_d \, K_c = 0.958 \times 1 = 0.958$$

$$T_s = \frac{S_s \, Z_{ss}}{6} = \frac{48 \times 30}{6} = 240$$

Equivalent resistance of rotor per phase referred to stator

$$r_r' = 4 \, m_s \, T_s^2 \, K_{ws} \, \rho \left[\frac{L_b}{S_r \, a_b} + \frac{2}{\pi} \, \frac{D_e}{P^2 a_e} \right]$$

$$r_r' = 4 \times 3 \times 240^2 \times 0.958^2 \times 0.021 \left[\frac{0.12}{53 \times 60} + \frac{2}{\pi} \, \frac{0.18}{4^2 \times 150} \right]$$

$$\boxed{r_r' = 1.14 \; \Omega}$$

EXAMPLE: 02

A 6 pole, 3 phase squirrel cage induction motor has 72 stator slots with 15 conductors in each slot. There are 55 rotor slots. The coil span is 11 slots and the phase spread is 60°.

Determine the current in rotor bars and in end rings if the stator current is 24.1 A and the power factor is 0.83.

Given data

$$P = 6 \qquad S_s = 72 \qquad Z_{ss} = 15 \qquad S_r = 55 \qquad \text{coil span} = 11$$

phase spread $m\beta = 60 \qquad I_s = 24.1 \, A \qquad \cos\phi = 0.83$

$$m = \frac{\text{slots}}{\text{poles} \times \text{phases}} = \frac{72}{6 \times 3} = 4$$

Distribution factor $K_d = \dfrac{\sin\left(\frac{m\beta}{2}\right)}{m \sin\left(\frac{\beta}{2}\right)} = \dfrac{\sin\left(\frac{60}{2}\right)}{4 \sin\left(\frac{60}{4 \times 2}\right)} = 0.958$

The number of slots per pole is $72/6 = 12$ and the coil span is 11 slots. Therefore, the coil are chorded by one slot pitch.

Angle of chording $\alpha = \frac{1}{12} \times 180 = 7.5$

pitch factor $K_p = \cos \frac{7.5}{2} = 0.998$

∴ winding factor $K_{ws} = 0.958 \times 0.998 = 0.956$

$$I_r' = I_s \cos\phi = 24.1 \times 0.83 = 20 \, A$$

Stator turns per phase $T_s = \dfrac{S_s \, Z_{ss}}{6} = \dfrac{72 \times 15}{3 \times 2} = 180$

$$I_b = \frac{2 \, m_s \, K_{ws} \, T_s}{S_r} \, I_r' = \frac{2 \times 3 \times 0.956 \times 180}{55} \times 20$$

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$$\boxed{I_b = 375.4 \text{ A}}$$

current in each end ring $I_e = \dfrac{S_r \, I_b}{\pi P} = \dfrac{55 \times 375.4}{\pi \times 6}$

$$\boxed{I_e = 1095.3 \text{ A}}$$

EXAMPLE: 03

Calculate the equivalent resistance of rotor per phase in terms of stator, current in each bar and end ring and total rotor $I^2 R$ loss for the following :

4 pole 3 phase, 50 Hz, 400 V cage motor has 48 slots in stator with 35 conductors per slot. Each conductor carries a current of 10 A. The rotor has 57 slots, each slot has a bar of 0.12 m length and 50 mm² area. The mean diameter of each ring is 0.2 m and area is 175 mm². Resistivity is 0.02 Ω/m and mm² and the power factor is 0.8. The stator winding uses full pitched coils with a phase spread of 60°.

Given Data

$P = 4 \qquad E_L = 400 \text{ V} \qquad S_s = 48 \qquad Z_{ss} = 35 \qquad I_s = 10 \text{ A}$

$S_r = 57 \qquad L_b = 0.12 \text{ m} \qquad a_b = 50 \text{ mm}^2 \qquad D_e = 0.2 \text{ m} \qquad a_e = 175 \text{ mm}^2$

$P = 0.02 \, \Omega/\text{m and mm}^2 \qquad \cos\phi = 0.8 \qquad m\beta = 60$

Full pitched coil

$$m = \frac{\text{slots}}{\text{poles} \times \text{phases}} = \frac{48}{4 \times 3} = 4$$

Distribution factor $K_d = \dfrac{\sin\left(\frac{m\beta}{2}\right)}{m \sin\left(\frac{\beta}{2}\right)} = \dfrac{\sin\left(\frac{60}{2}\right)}{4 \sin\left(\frac{60}{4\times 2}\right)} = 0.958$

For full pitched coil, pitch factor $K_p = 1$

$\therefore K_{ws} = K_d \, K_p = 0.958 \times 1 = 0.958$

$I_r' = I_s \cos\phi = 10 \times 0.8 = 8 \text{ A}$

$T_s = \dfrac{S_s \, Z_{ss}}{6} = \dfrac{48 \times 35}{6} = 280$

current in each bar $I_b = \dfrac{2 m_s \, K_{ws} \, T_s}{S_r} \, I_r' = \dfrac{2 \times 3 \times 0.958 \times 280}{57} \times 8$

$$\boxed{I_b = 226 \text{ A}}$$

current in each ring $I_e = \dfrac{S_r \, I_b}{\pi P} = \dfrac{57 \times 226}{\pi \times 4} = 1024 \text{ A}$

$$\boxed{I_e = 1024 \text{ A}}$$

Resistance of each bar $= \dfrac{P \, L_b}{a_b} = \dfrac{0.02 \times 0.12}{50} = 48 \times 10^{-6} \, \Omega$

$$I^2R \text{ loss in bars} = S_\gamma \, I_b^2 \, r_b = 57 \times (226)^2 \times 48 \times 10^{-6} = 140 \, W$$

$$\text{Resistance of each ring} = \rho \, \frac{\pi \, D_e}{a_e} = \frac{0.02 \, \pi \times 0.2}{175} = 72 \times 10^{-6} \, \Omega$$

$$I^2R \text{ loss in 2 rings} = 2 \, I_e^2 \, r_e = 2 \times (1024)^2 \times 72 \times 10^{-6} = 151 \, W$$

$$\boxed{\text{Total rotor } I^2R \text{ loss} = 140 + 151 = 291 \, W}$$

$$r_\gamma' = \frac{\text{Total rotor } I^2R \text{ loss}}{m_s \, I_\gamma'^2} = \frac{298}{3 \times 8^2} = 1.51 \, \Omega$$

Equivalent resistance of rotor per phase in terms of stator

$$\boxed{r_\gamma' = 1.51 \, \Omega}$$

CIRCLE DIAGRAM

It is possible to obtain graphically a considerable range of information from circle diagra The construction gives estimate of full load current and power factor, maximum power outpr pull out torque and the full load efficiency and slip. The circle diagram is constructed from t following design data :

I_m = magnetizing current phase,
I_l = loss component of no load current per phase,
X_s = total standstill leakage reactance per phase referred to stati
R_s = total resistance per phase referred to stator,
Z_s = total short circuit impedance per phase referred to stator,
E_s = stator voltage per phase.

The procedure for drawing the circle diagram is given below

Circle diagram.

1. Draw Oa and Ob perpendicular to each other.
2. Draw $OO' = I_0$, the no load current per phase at an angle ϕ_g with Ob after choosing suitable current scale.

$$\phi_0 = \tan^{-1} I_m/I_l \text{ and } I_0 = \sqrt{I_m^2 + I_l^2}.$$

3. Draw $O'D$ passing through O' and parallel to Oa.
4. Draw $OB = I_{scs}$ short circuit current per phase at an angle ϕ_{sc}.

$$\phi_{sc} = \tan^{-1} X_s/R_s \text{ and } I_{sc} = E_s/Z_s.$$

5. Join O' with B.

6. Construct the perpendicular bisector of $O'B$ intersecting the line $O'D$ at C. Point C is the centre of circle having radius $O'C$.

7. Draw the circle $O'BD$.

8. Draw BF perpendicular to $O'D$ and divide it at G in such a way that

$$\frac{BG}{GF} = \frac{\text{rotor resistance referred to stator}}{\text{stator resistance}} = \frac{r_r'}{r_s}.$$

9. Join O' to G.

The line $O'G$ is known as torque line and line $O'B$ as output line. The diagram can be used to determine the characteristics for any current.

10. In design, we are mainly interested in characteristics at rated output. The point A corresponding to rated output can be located as given below :

Suppose we have drawn the diagram with a current scale of 1 cm = x ampere.

$\therefore$ 1 cm = $x\,E_s$ watt per phase.

(It should be borne in mind that we have drawn the diagram considering phase current and phase voltage and therefore the results are obtained in terms of power per phase. In order to get total power we will have to multiply the results for power and torque by 3 for 3 phase machines).

Extend line FB Cut off BS = rated output per phase.

Draw a line SA parallel to output line $O'B$ cutting the circle at A. Then A is the operating point for rated output.

11. Draw AH perpendicular to Oa.

12. Join O' to A. This gives : ϕ_r = rotor current phase angle.

13. Label points JKL.

Stator current per phase at full load $I_s = OA$.

Stator power factor at full load $\cos\phi = AH/OA$.

Constant loss   = 3 × JH.

Rotor copper loss at full load   = 3 × LK.

JH and LK are measured on power scale.

Slip $s = \dfrac{\text{rotor copper loss}}{\text{rotor input}} = \dfrac{LK}{AK}.$

Efficiency $\eta = \dfrac{\text{rotor output}}{\text{stator input}} = \dfrac{AL}{AH}.$

Torque = 3 × AK W (synchronous)

with AK measured on power scale.

The location of point M on circle for maximum power output is done by drawing a perpendicular on the output line from C. Line MN represents maximum output.

Maximum output   = 3 × MN.

The location of point P on circle for maximum torque is done by drawing a perpendicular on torque line from C. Line PQ represents the maximum torque.

Maximum torque   = 3 × PQ W (synchronous).

Line BG represents starting torque. Starting torque = 3 × BG.

MN, PQ and BG are measured on power scale.

The values of slip, efficiency, and power factor can be more accurately determined by using some additional graphical methods.
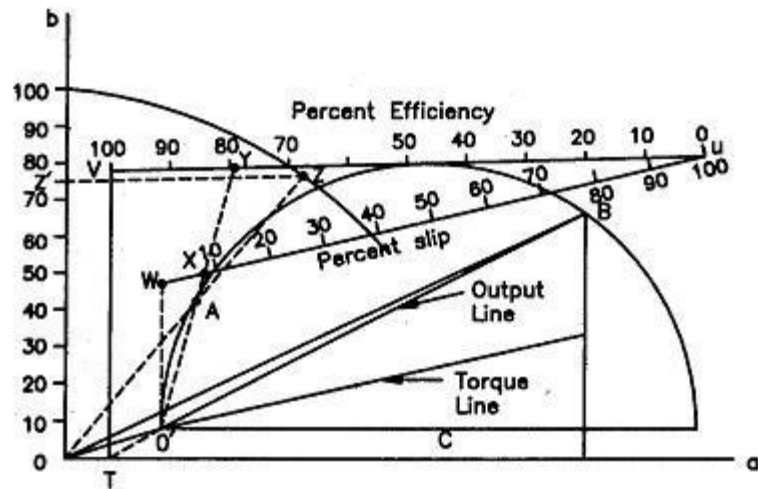
Determination of slip, efficiency and power factor from circle diagram.

Fig   shows the circle diagram drawn as per the procedure given above.

Draw the output line O'B backwards to ent Oa at T.

Draw vertical lines TV and O'W.

Extend output line O'B to a convenient point U.

From U, draw line UW parallel to torque line and line UV parallel to horizontal axis Oa.

Divide both lines Ob and UW into 100 parts.

Extend O'A to cut line UW at X. Point X indicates the value of percentage slip.

Extend O'A to cut line UV at Y. Point Y indicates the percentage efficiency.

The power factor is easily found by drawing a quadrant of a circle with O as center and a radius 100 arbitrary units.

Extend OA to intersect the quadrant at Z.

Project OZ on vertical axis Qb (this is OZ')

Vertical projection OZ' gives the value of power factor.

TWO MARKS QUESTION AND ANSWERS

01   What is rotating transformer?

The principle of operation of induction motor is similar to that a transformer. The stator winding is equivalent to primary of a transformer and the rotor winding is equivalent to short circuited secondary of a transformer. In transformer the secondary is fixed but in induction motor it is allowed to rotate. Hence the induction motor is also called rotating transformer.

02   What are the advantages of cage induction motor over slip ring induction motor?

The advantages of squirrel cage motor are

- It is cheaper than slip ring motors
- It does not have any wear and tear parts like slip rings, brush gear and short circuiting devices. Hence the construction will be rugged.
- The rotor slots can be fully occupied by the conductor due to absence of insulation. Hence the rotor bars may have low resistance. Also there is no overhang (or small overhang) in rotor winding. Due to these two factors the rotor copper loss will be lesser than slip ring motors and so efficiency will be slightly higher.
- Due to smaller overhang leakage reactance, the motor will have better power factor, a greater pull out torque and over load capacity.

03   What are the advantages of slip-ring rotor over cage rotor?

The advantages of slip ring rotor are

- The starting torque can be varied by adding resistance to rotor.
- The speed of the machine can be varied by injecting an emf through slip rings to the rotor.

04   Write the expression for output equation and output coefficient of induction motor.

The equation for input kVA is considered as output equation in induction motor.
The input kVA, $Q = C_o D^2 L n_s$ in kVA
Output coefficient, $C_o = 11 K_{ws} B_{av} \, ac \times 10^{-3}$ in kVA/m^3-rps.

05   What are the factors to be considered for the choice of specific magnetic loading?

The choice of specific magnetic loading depends on power factor, iron loss and over load capacity.

06   What are the factors to be considered for the choice of specific electric loading?

The choice of specific electric loading depends on copper loss, temperature rise, voltage rating and overload capacity.

07   How the induction motor can be designed for best power factor?

For best power factor the pole pitch, τ is choesn such that, $\tau = \sqrt{0.18L}$.

08   Write the expression for length of mean turn of stator winding.

Length of mean turn of stator, $L_{mts} = 2L + 2.3 \, \tau + 0.24$
where, L = Length of stator and λ = Pole pitch.

09 Why short chorded windings are employed in induction motor?

For short chorded windings the length of mean turn will be lesser than the full pitch coils. Hence it results in reduction of copper. Also the short chorded windings eliminates certain harmonic magnetic fields.

10 What type of slots are preferred in induction motor?

Semienclosed slots are preferred for induction motor. It results in less air gap contraction factor giving a small value of magnetizing currents, low tooth pulsation loss and much quieter operation (less noise).

11 What are the factors to be considered for selecting the number of slots in induction machines stator?

The factors to be considered for selecting the number of slots are tooth pulsation loss, leakage reactance, magnetizing current, iron loss and cost. Also the number of slots should be multiple of slots per pole per phase for integral slot winding.

12 What are the advantages and disadvantages of large air-gap length in induction motor?

Advantage : A large air-gap length results in higher overload capacity, better cooling, reduction in noise and reduction in unbalanced magnetic pull.

Disadvantage : The disadvantage of large air-gap length is that it results in high value of magnetizing current.

13 What are the criteria used for the choice of number of slots of an induction machine?

The factors to be considered for the choice of number of stator slots of an induction machine are slot loading, slot pitch, type of winding and harmonic torques.

Slot loading	:	Slot loading should not exceed 750 amp.cond.
Slot pitch	:	The slot pitch should lie between 15 mm to 25 mm.
Type of winding	:	1. For integral slot winding the stator slots should be a multiple of slots per pole per phase.
		2. For double layer winding, the conductors per slot should be even.
Harmonic Torque	:	Certain combinations of stator and rotor slots give rise to harmonic torques which results in crawling and cogging. To avoid these undesirable effects the difference between stator and rotor slots should not be equal to 0, ±1, ±2, $\pm p$, $\pm2p$, $\pm3p$, $\pm5p$, $\pm(p \pm 1)$, $\pm(p \pm 2)$.

14 What are the factors to be considered for estimating the length of air-gap in induction motor?

The following factors are to be considered for estimating the length of air-gap.

- Power factor
- Overload capacity
- Pulsation loss
- Unbalanced magnetic pull
- Cooling
- Noise

15 List out the methods to improve the power factor of induction motor.

The power factor of the induction motor can be improved by reducing the magnetizing current and leakage reactance.

The magnetizing current can be reduced by reducing the length of air-gap. The leakage reactance can be reduced by reducing the depth of stator & rotor slots, by providing short chorded winding and reducing the overhang in stator winding.

16 Why the air-gap of an induction motor is made as small as possible?

The mmf and the magnetizing current are primarily decided by length of air-gap. If air-gap is small then mmf and magnetizing current will be low, which in turn increase the value of power factor. Hence by keeping small air-gap, high power factor is achieved.

17 Write the formula for air-gap in case of three phase induction motor in terms of length and diameter.

$$\text{The length of air-gap, } l_g = 0.2 + 2\sqrt{DL} \text{ in mm}$$

where D and L are expressed in metre.

18 Discuss the relative merits of open and closed slots for induction motor rotor.

The closed slots will not increase reluctance of air-gap and has lesser noise but it has difficulty in casting the rotor bars.

The open slots increase the reluctance of air-gap and has high noise but it offers flexibility in casting rotor bars.

19 List the undesirable effects produced by certain combination of rotor and stator slots.

The following problems may develop in induction motor with certain combination of rotor and stator slots.

- The motor may refuse to start (cogging).
- The motor may run at subsynchronous speed (crawling).
- Severe vibrations may develop and the noise will be excessive.

20 What is crawling and cogging?

Crawling is a phenomena in which the induction motor runs at a speed lesser than subsynchronous speed.

Cogging is a phenomena in which the induction motor refuse to start.

21 What is skewing?

Skewing is twisting either the stator or rotor core. The motor noise, vibrations, cogging and synchronous cusps can be reduced or even entirely eliminated by skewing.

In order to eliminate the effect of any harmonic, the rotor bars should be skewed through an angle so that the bars lie under alternate harmonic poles of the same polarity or in other words the bars must be skewed through two pitches.

22   Explain the phenomena of cogging.
When the number of stator and rotor slots are equal, the speeds of all the harmonics produced by stator slotting coincide with the speed of corresponding, rotor harmonics. Thus harmonics of every order would try to exert synchronous torques at their corresponding synchronous speeds and the machine would refuse to start. This is known as cogging.

23   State the effect of change of air-gap length in a three phase induction motor

If the air-gap of an induction motor is changed then the mmf and magnetizing current also changes. i.e. Increase in air-gap length increases the overload capacity, offers better cooling, reduces noise and reduces unbalanced magnetic pull.

24   Define unbalanced magnetic pull.

The unbalanced magnetic pull is the radial force acting on the rotor due to non uniform air-gap around armature periphery.

25   How does the external resistance of slip-ring induction motor influence the motor performance.

External resistance connected to slip-ring

1. Increases the starting torque
2. Decreases the starting current   and
3. Used to control the speed of rotor.

26   State the main constructional differences between cage induction motor and slip-ring induction motor.

Squirrel Cage Induction Motor	Slip-ring Induction Motor
Slip rings and brushes are absent.	Slip-rings and brushes are present to add external resistances.
Rotor consists of bars which are shorted at the ends with the help of end rings.	Rotors consists of a three phase winding similar to the stator winding.
The rotor automatically adjusts itself for the same number of poles as that of stator.	Rotor must be wound for the same number of poles as that of stator.

27   What are the different losses in an induction motor.

a)   Rotational or constant losses   b) I^2R losses or variable losses
Rotational losses are made up of: i) Friction and windage losses   ii) Iron losses
I^2R losses are made up of   :   i) Stator cu loss   ii) Rotor cu loss

28   List the main parts of a slip-ring Induction motor.

1. Stator   2. Rotor   3. Slip rings   4. Metal collar
5. Brushes   6. Bearings   7. Fan

29   Write an empirical formula for finding the length of the air gap of an induction motor.
$$l_g = 0.2 + 2\sqrt{DL}$$
Where   D = Diameter of bore
           L = Length of stator   D & L are expressed in metre.

30   How do you select L/τ ratio for design of induction motor.

L/τ ratio is selected based on design feature.

Design Feature	Ratio L/τ
(1) Minimum cost	1.5 to 2
(2) Good Power Factor	1 to 1.25
(3) Good Efficiency	1.5
(4) Good Overall design	1

Generally L/τ lies between 0.6 to 2. It can be shown that for best power factor the pole pitch τ is given by the equation $\tau = \sqrt{0.18L}$ ...

31   Why the length of air gap in an induction motor is kept minimum possible range.

The mmf and the magnetizing current are primarily decided by length of air-gap. If air-gap is small then mmf and magnetizing current will be low, which in turn increase the value of power factor. Hence by keeping small air-gap, higher power factor is achieved.

32   What factors govern the choice of air gap in induction motor.

The factors govern the choice of air gap in induction motor.
1. Power factor   2. Unbalanced magnetic pull   3. Overhead capacity
4. Pulsation loss   5. Noice   6. Cooling

33   Define dispersion coefficient.

The dispersion coefficient is defined as the ratio of magnetizing current to ideal short circuit current.

Dispersion coefficient, $\sigma = I_m / I_{sci}$

where, I_m = Magnetizing current
         I_{sci} = E_s/X_s = Ideal short circuit current
         E_s = Stator phase voltage
         X_s = Total leakage reactance of the motor referred to stator